Name: ______________________________ Date: ________________

Review - Completing the Square

Completing the Square

For the quadratic $y = x^2 + bx + c$ (note that $a = 1$).

1. Find half of the value of b, i.e. $\frac{b}{2}$
2. Square it, i.e. $\left(\frac{b}{2}\right)^2$
3. Add and subtract it

Exercise #6: Write each quadratic in vertex form by Completing the Square. Then, identify the quadratic's turning point.

(a) $y = x^2 + 6x - 2$

(b) $y = x^2 - 2x + 11$

(c) $y = x^2 - 10x + 27$

(d) $y = x^2 + 8x$

(e) $y = x^2 + 5x + 4$

(f) $y = x^2 - 9x - 2$

Completing the Square, when the leading coefficient doesn't equal 1, is much more difficult to master and to understand. Always remember that you are writing an **equivalent expression** by essentially **adding zero** in one way or another.

Exercise #7: Consider the quadratic $y = 2x^2 - 12x + 11$.

(a) $y = 5x^2 + 20x + 23$

(b) $y = -2x^2 + 4x + 7$

(c) $y = 6x^2 - 24x + 14$

(d) $y = -x^2 - 12x - 33$

Name: ________________________ Date: ______________

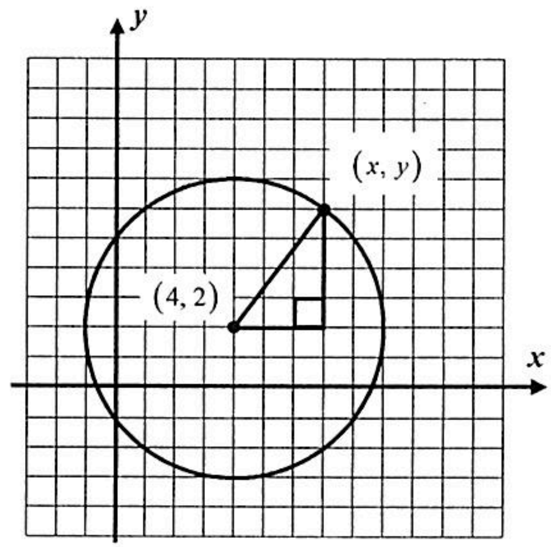
6-4 EQUATIONS OF CIRCLES

Various quadratic relationships can be placed into equations by knowing the **locus definition** of the relationship. We will explore this for parabolas in a future lesson. In this one, we will develop the **equation** of a **circle** by using the **distance formula** that you learned from Common Core Geometry.

THE DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) is given by: $D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

***Exercise* #1:** A circle is the collection of all points that are a set distance (the radius) away from a point (its center). The circle shown below has a radius of 5 and a center at the point $(4, 2)$. An arbitrary point on the circle, (x, y), is shown marked.



(a) Using the distance formula show that the point $(7, -2)$ must lie on this circle (verify graphically).

(b) Letting $(x_2, y_2)=(x, y)$ and $(x_1, y_1)=(4, 2)$, write the distance formula for all points on this circle.

(c) Square both sides of the equation from (b) to create the standard form of a circle.

(d) Show algebraically that the point $(1, -2)$ must also lie on the circle.

THE EQUATION OF A CIRCLE

A circle whose center is at (h, k) and whose radius is r is given by: $(x-h)^2+(y-k)^2=r^2$

***Exercise* #2:** Which of the following equations would have a center of $(-3, 6)$ and a radius of 3?

(1) $(x-3)^2+(y+6)^2=9$ (3) $(x-3)^2+(y-6)^2=3$

(2) $(x+3)^2+(y-6)^2=9$ (4) $(x+3)^2+(y+6)^2=3$

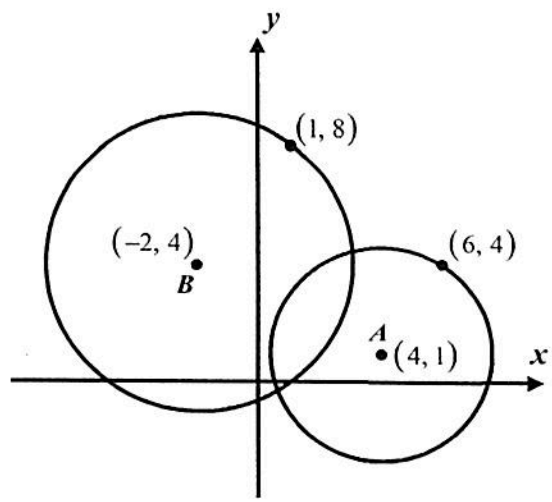
Exercise #3: For each of the following equations of circles, determine both the circle's center and its radius. If its radius is not an integer, express it in decimal form rounded to the nearest *tenth*.

(a) $(x-2)^2+(y-7)^2=100$ (b) $(x-5)^2+(y+8)^2=4$ (c) $x^2+y^2=121$

(d) $(x+1)^2+(y+2)^2=1$ (e) $x^2+(y-3)^2=49$ (f) $(x+6)^2+(y-5)^2=18$

(g) $x^2+y^2=64$ (h) $(x-4)^2+(y-2)^2=20$ (i) $x^2+y^2=57$

Exercise #4: Write equations for circles A and B shown below. Show how you arrive at your answers.



Exercise #5: By completing the square on both quadratic expressions in x and y determine the center and radius of a circle whose equation is

$x^2+10x+y^2-2y=10$ $x^2-6x+y^2+10y=66$

Use the information provided to write the standard form equation of each circle.

1) $8x + x^2 - 2y = 64 - y^2$

2) $137 + 6y = -y^2 - x^2 - 24x$

3) $x^2 + y^2 + 14x - 12y + 4 = 0$

4) $y^2 + 2x + x^2 = 24y - 120$

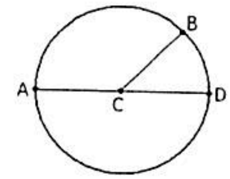
5) $x^2 + 2x + y^2 = 55 + 10y$

6) $8x + 32y + y^2 = -263 - x^2$

6-5 Arcs, Chords and Central Angles

A ___________ angle is an angle whose vertex is the ___________ of a circle.

In this diagram ∠_____ and ∠ _____ are central angles.

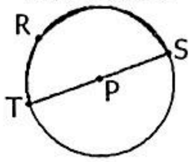


An ______ is part of a circle.

There are different types of arcs:

Semicircle

half a circle

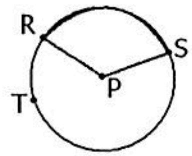


______ is a semicircle

m _______ = ______

Minor Arc

smaller than a semicircle

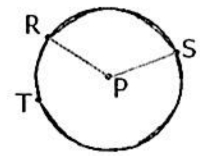


______ is a minor arc

m _____ = m ∠ _____

Major Arc

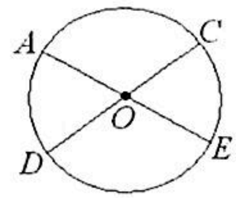
larger than a semicircle



______ is a major arc

m _____ = 360 - m _____

Example 1: Identify the following in circle O:

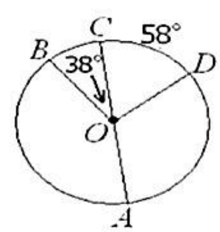


a) the minor arc(s): ______________________________

b) the semicircle(s): _________________________

c) the major arcs that contain point A: ___________________

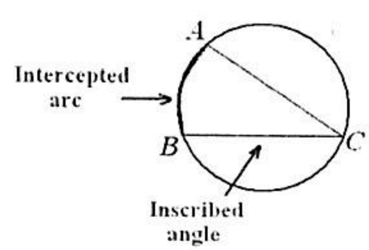
Example 2: Find the measure of each arc:



a) m $\widehat{BC}$ = ______ b) m $\widehat{BD}$ = ______

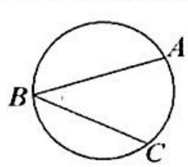
c) m $\widehat{ABC}$ = _______ d) m $\widehat{AB}$ = ______

INSCRIBED ANGLE:



- Vertex is _____________________________
- Sides are _______________.

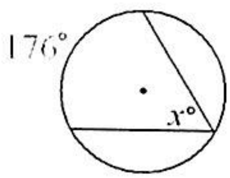
The measure of an inscribed angle is half the measure of its intercepted **arc.**



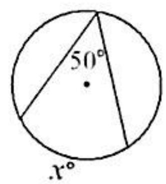
$$m\angle B = \frac{m\widehat{AC}}{2}$$

Example 1: Find the measure of the variable in each diagram:

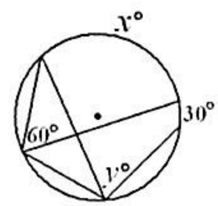
a)



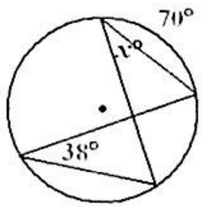
b)



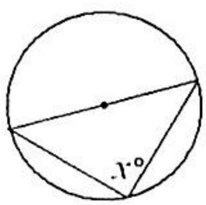
c)



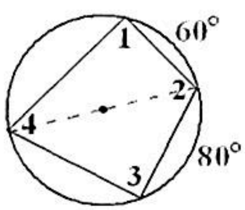
Two inscribed angles that intercept the same arc are ________________.



An angle inscribed in a semicircle is a ________________.



The opposite angles of a quadrilateral inscribed in a circle are ______________________________.

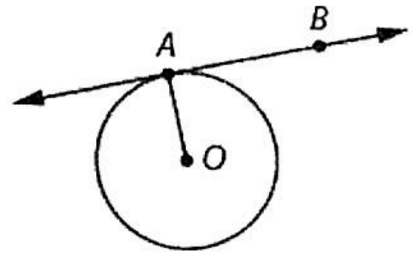


Tangents to a Circle

As you learned in Math 2, lines that touch a circle in exactly one point are said to be ________________ to the circle.

Tangent at a Point on a Circle:

- Center point: ______
- Radius: ________
- Tangent line: ________
- Point of tangency: _____



Relative to r, how long is $\overline{OA}$? _______

Is B in the interior or exterior of circle O? ____________ Relative to r, how long is $\overline{OB}$? ___________

Draw point C in the interior of circle O. Relative to r, how long is $\overline{OC}$? ____________

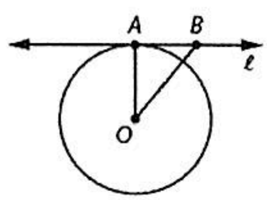
What kind of angle does ∠OAB appear to be?

Theorem: If a line is tangent to a circle, then it is ______________________ to the radius at the point of tangency.

Examples:

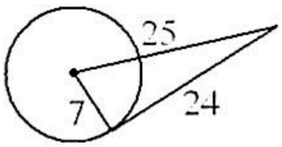
1. If line ℓ is tangent to a circle O at point A, the radius of the circle is 4 in., and AB = 3 in., what is length BO? Explain.

2. If AB = 5 cm, AO = 12 cm, and BO = 13 cm, why is it correct to conclude that line ℓ must be tangent to the circle at point A?

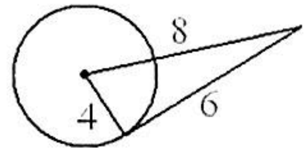


3. Is there a tangent line?

a)

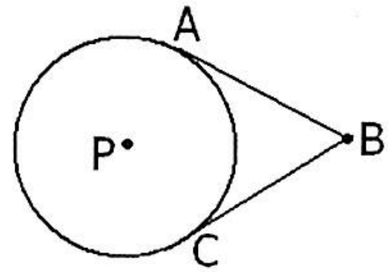


b)



Tangents from a Point not on a Circle:

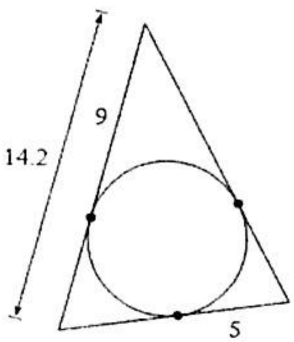
Suppose B is a point on the exterior of Circle P. Suppose $\overline{AB}$ and $\overline{CB}$ are tangents to Circle P. How could you use the Pythagorean Theorem to show that $\overline{AB} \cong \overline{CB}$?



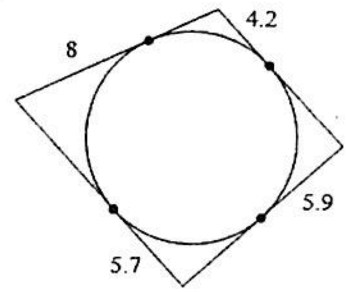
Applying Properties of Tangents:

Find the perimeter of the following shapes:

1)



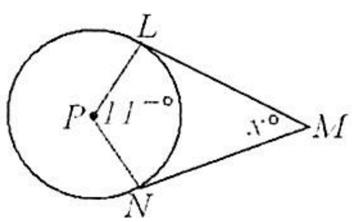
2)



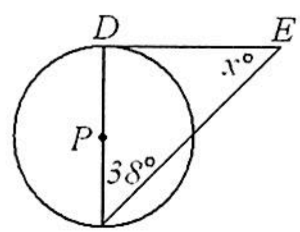
EXAMPLES!

Ex.1: Finding Angle Measures

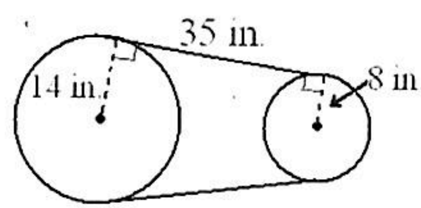
a.)



b.)

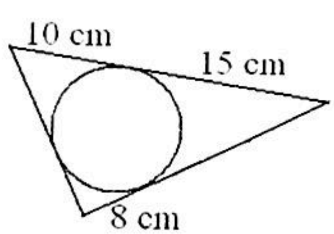


Ex.2: A belt fits tightly around two circular pulleys. Find the distance between the centers of the pulleys.

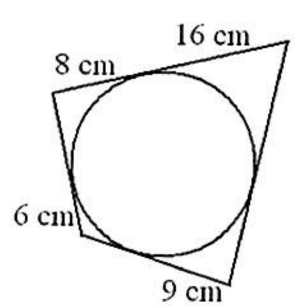


Ex.3: Each polygon circumscribes a circle. Find the perimeter of the polygon.

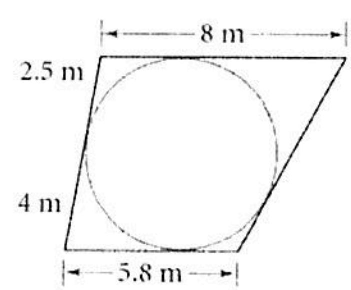
a.)



b.)

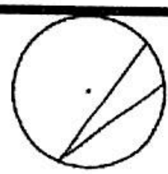


c)

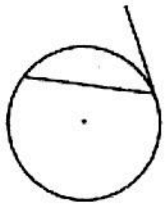


ON the circle:

Inscribed Angle

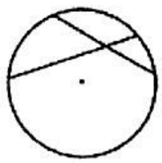


Tangent-Chord Angle

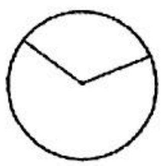


INSIDE the circle:

Chord-Chord Angle

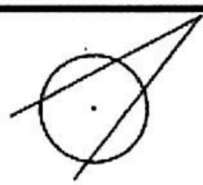


Central Angle

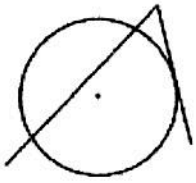


OUTSIDE the circle:

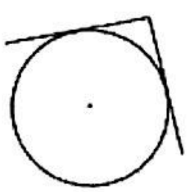
Secant-Secant Angle



Secant-Tangent Angle

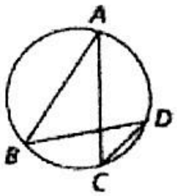


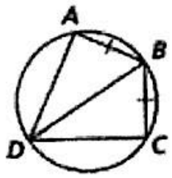
Tangent-Tangent Angle

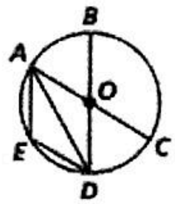


Angles & Circles:

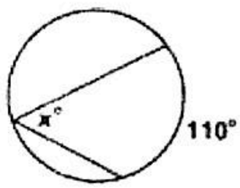
For each diagram, indicate a pair of congruent inscribed angles.

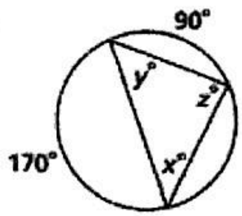
1. 

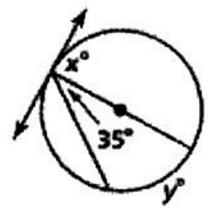
2. 

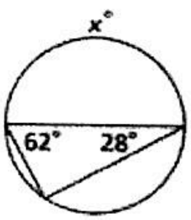
3. 

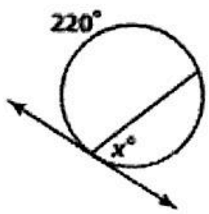
Find the value of each variable.

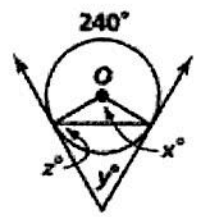
4. 

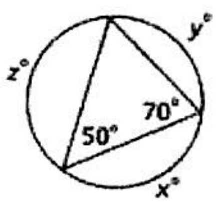
5. 

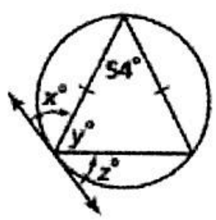
6. 

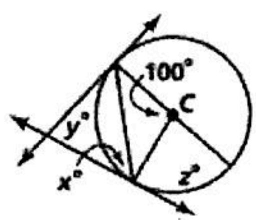
7. 

8. 

9. 

10. 

11. 

12. 

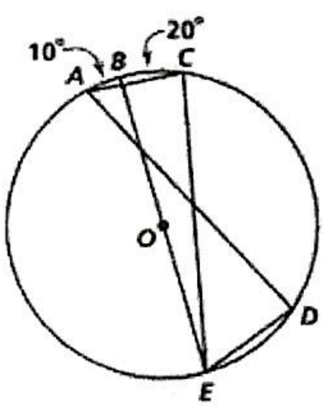
Find each indicated measure for ⊙*O*.

13. a. $m\widehat{AE}$

b. $m\angle C$

c. $m\angle BEC$

d. $m\angle D$

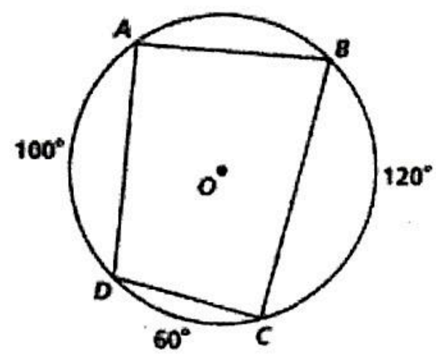


14. a. $m\angle A$

b. $m\angle B$

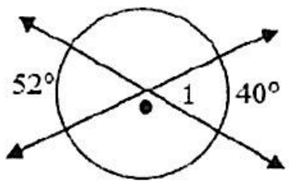
c. $m\angle C$

d. $m\angle D$

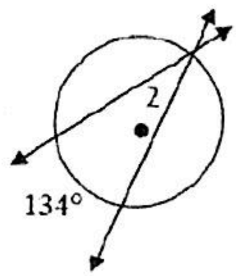


Find the measure of each numbered angle.

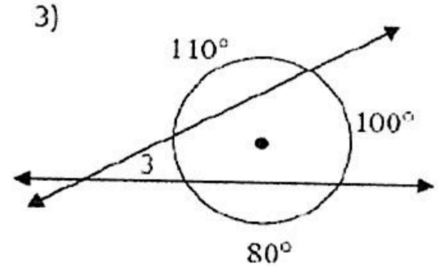
1)



2)

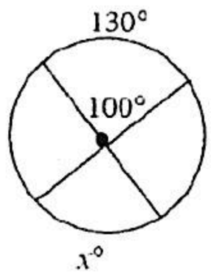


3)

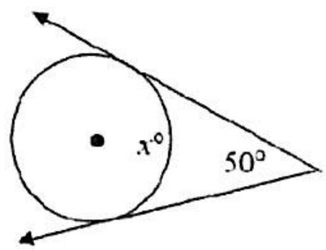


Find the value of x.

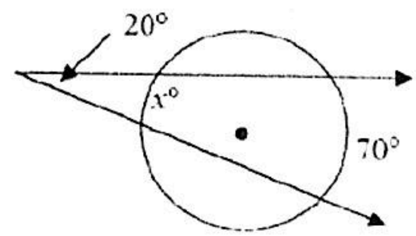
4)



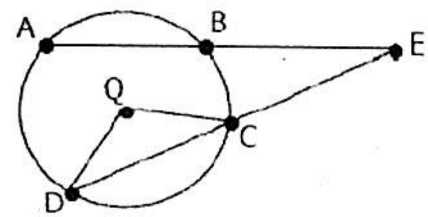
5)



6)



Assume that lines that appear to be tangents are tangents. In $\odot Q$, $m\angle CQD = 120°$, $m\widehat{BC} = 30°$, and $m\angle BEC = 25°$. Find each measure.



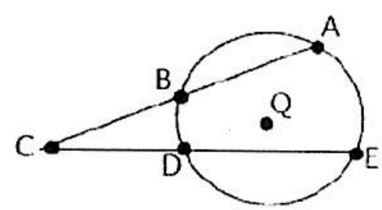
7) $m\widehat{DC}$

8) $m\widehat{AD}$

9) $m\widehat{AB}$

10) $m\angle QDC$

In $\odot Q$, $m\widehat{AE} = 140°$, $m\widehat{BD} = y°$, $m\widehat{AB} = 2y°$, and $m\widehat{DE} = 2y°$. Find each measure.



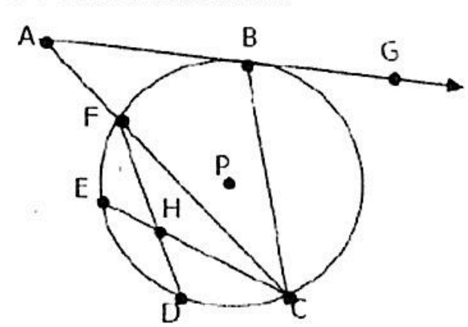
11) $m\widehat{BD}$

12) $m\widehat{AB}$

13) $m\widehat{DE}$

14) $m\angle BCD$

In $\odot P$, $m\widehat{BC} = 4x - 50$, $m\widehat{DE} = x + 25$, $m\widehat{EF} = x - 15$, $m\widehat{FB} = 50$, and $m\widehat{CD} = x$. Find each measure.



15) $m\angle A$

16) $m\angle BCA$

17) $m\angle ABC$

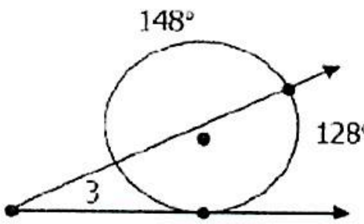
18) $m\angle GBC$

19) $m\angle FHE$

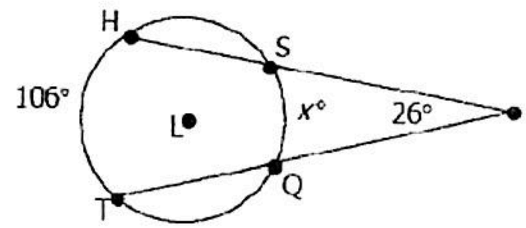
20) $m\angle CFD$

Use the diagram to find the missing information.

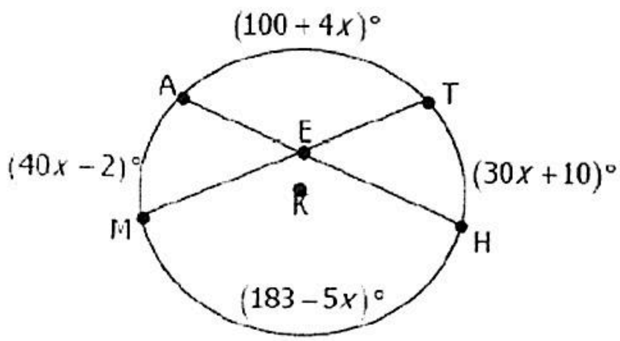
21) Find $m\angle 3$



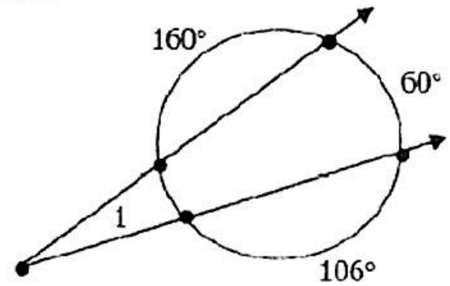
22) Find the value of x.



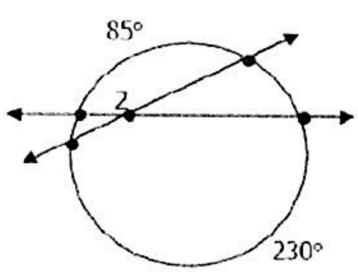
23) Find the value of x and $m\angle AET$.



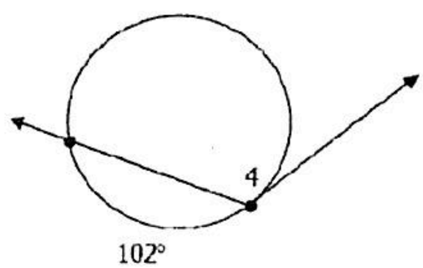
24) Find $m\angle 1$.



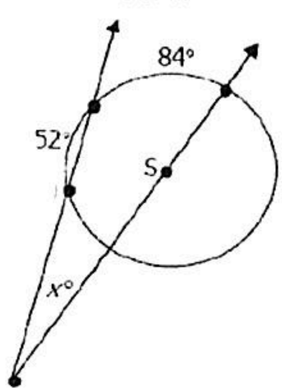
25) Find $m\angle 2$,



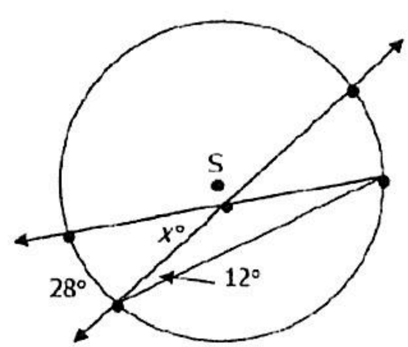
26) Find $m\angle 4$.



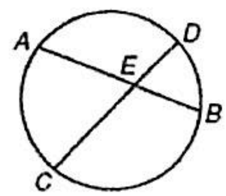
27) Find the value of x.



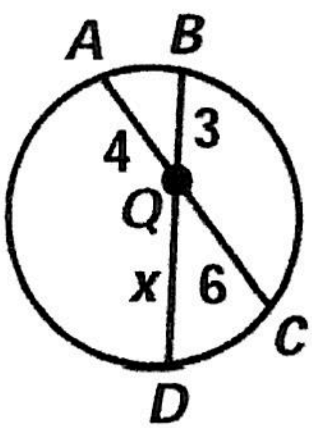
28) Find the value of x.

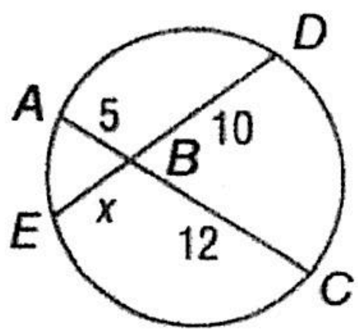


6-6 Circle Segment Theorems

Chord-Chord Product Theorem	
If two chords intersect in the interior of a circle, then the products of the lengths of the segments of the chords are equal.	 $AE \cdot EB = CE \cdot ED$

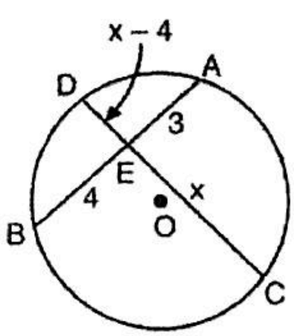
Level A: Find the value of x of each of the problems below:



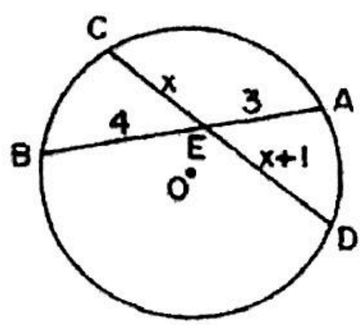


Level B:

In the accompanying diagram of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $AE = 3$, $EB = 4$, $CE = x$, and $ED = x - 4$, what is the value of x?



In the accompanying diagram of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $AE = 3$, $EB = 4$, $CE = x$, and $ED = x + 1$, find CE.

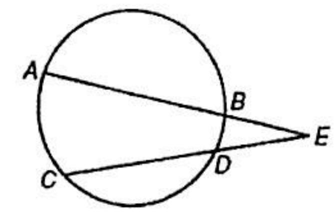


Secant-Secant Product Theorem

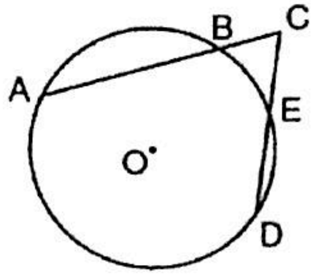
The product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

whole • outside = whole • outside

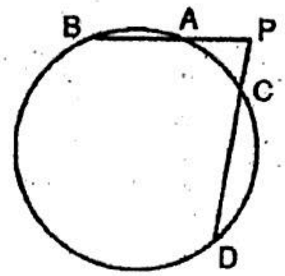
$AE \cdot BE = CE \cdot DE$



In the accompanying diagram of circle O, secants $\overline{CBA}$ and $\overline{CED}$ intersect at C. If $AC = 12$, $BC = 3$, and $DC = 9$, find EC.



In the diagram below, $\overline{PAB}$ and $\overline{PCD}$ are secants to the circle. If $PA = 4$, $AB = 5$, and $PD = 12$, what is PC?

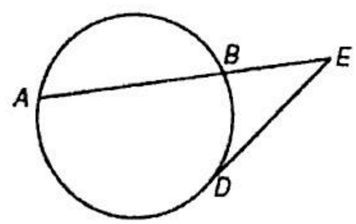


Secant-Tangent Product Theorem

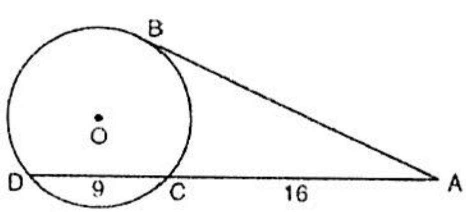
The product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared.

whole • outside = tangent^2

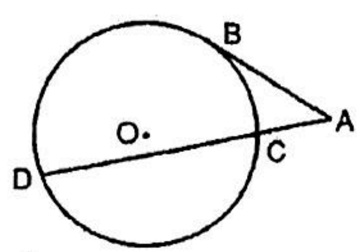
$AE \cdot BE = DE^2$



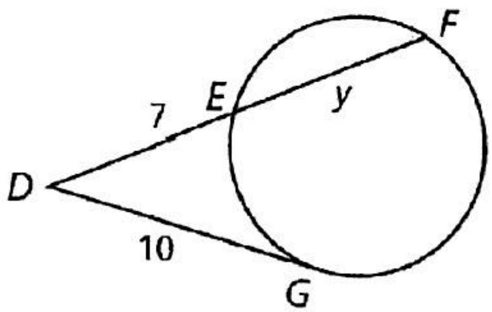
In the accompanying diagram $\overline{AB}$, is tangent to circle O at B. If $AC = 16$ and $CD = 9$, what is the length of $\overline{AB}$?

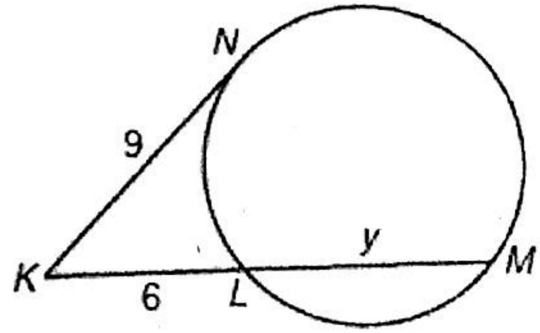


In the accompanying diagram, tangent $\overline{AB}$ and secant $\overline{ACD}$ are drawn to circle O from point A. If $AC = 4$ and $CD = 12$, find AB.



Find the value of *y*.

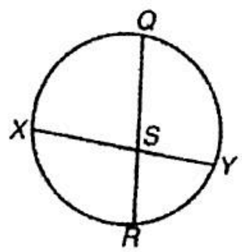




Summary

The models below show segment relationships in circles.

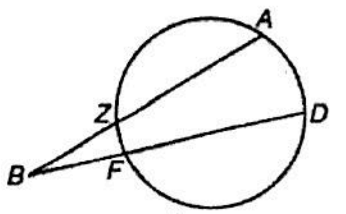
Chord-Chord



Chords $\overline{XY}$ and $\overline{QR}$ intersect at S.

$RS \cdot SQ = XS \cdot SY$

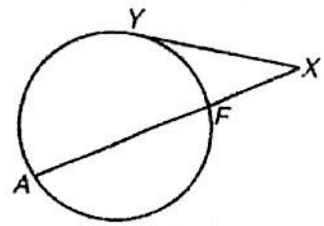
Secant-Secant



Secants $\overline{AB}$ and $\overline{DB}$ intersect at B.

$AB \cdot ZB = DB \cdot FB$

Secant-Tangent

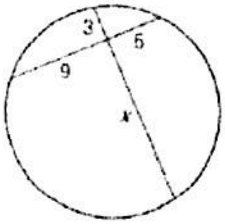


Secant $\overline{AX}$ and tangent $\overline{YX}$ intersect at X.

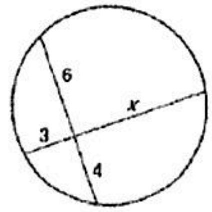
$AX \cdot FX = YX^2$

Fill in the blanks. Then find the value of x.

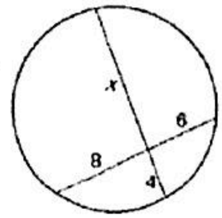
1) $x \cdot$ ________ $= 5 \cdot$ ________



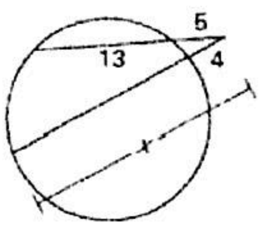
2) $6 \cdot$ ________ $= 3 \cdot$ ________



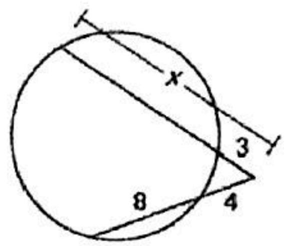
3) $x \cdot$ ________ $= 8 \cdot$ ________



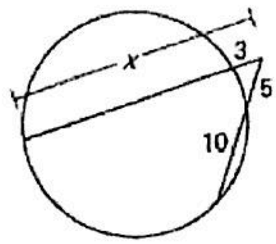
4) $4 \cdot$ ________ $= 5 \cdot$ ________



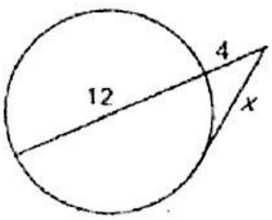
5) $3 \cdot$ ________ $= 4 \cdot$ ________



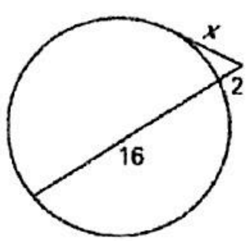
6) $3 \cdot$ ________ $= 5 \cdot$ ________



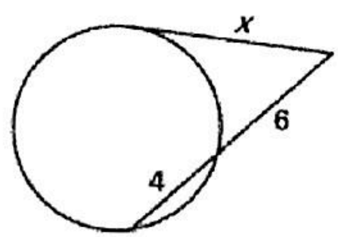
7) $x^2 = 4 \cdot$ ________



8) $x^2 = 2 \cdot$ ________

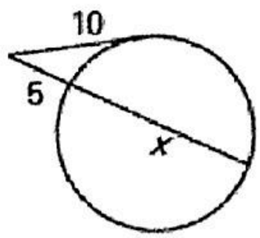


9) $x^2 = 6 \cdot$ ________

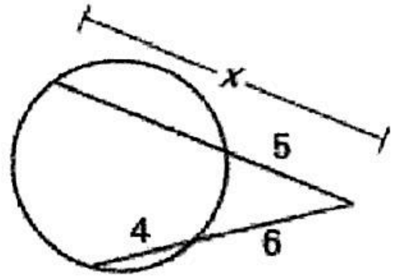


Find the value of x.

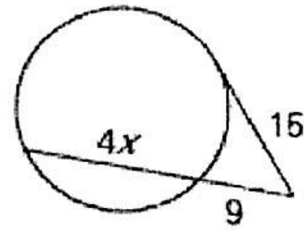
13) $x =$ ________



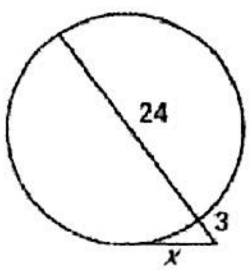
14) $x =$ ________



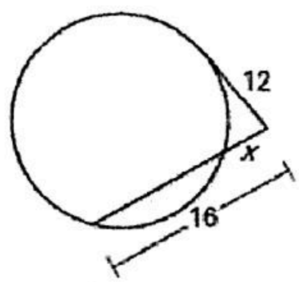
15) $x =$ ________



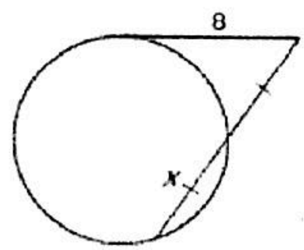
16) $x =$ ________



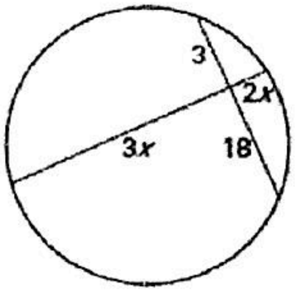
17) $x =$ ________



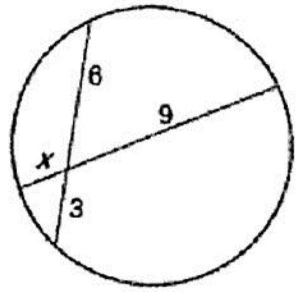
18) $x =$ ________



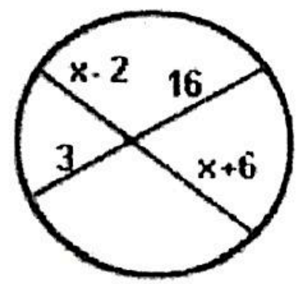
19) $x =$ ________



20) $x =$ ________



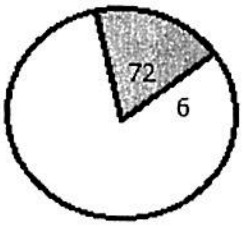
21) $x =$ ________



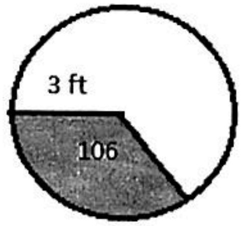
Arc Length and Sector Area

Find the arc length and sector area for the shaded area.

1)



2)



3)

4)