Unit 3 – Part II Logarithms

Exponentials and logarithms are _____________ functions: they " ___________" each other.

$$\text{If } y = b^x \text{, then } \log_b y = x$$

Do the LOOP!

<u>Examples</u>:

I. Write in exponential form:

1. $\log_3 81 = 4$

2. $\log_{36} 6 = ½$

3. $\log_3 1/9 = -2$

II. Write in logarithmic form:

4. $32 = 2^5$

5. $5^4 = 625$

6. $10^0 = 1$

III. Evaluate

To evaluate a log expression, convert to exponential form.

1. So $\log_2 8$ would be written:

   "Do the Loop" and solve!

2. So $\log_{27} 3$ would be written:

You try!

a. $\log_5 25 =$

b. $\log_{16} 4 =$

A ___________ logarithm is a log that uses base _____. You can drop the base 10 (assumed):

So " $\log_{10} y$ " can be written : $\log y$.

* Your calculator understands base 10 logs.

Evaluate using the LOG button in your calculator:

1. $\log 1000 =$

IV. Rewrite using **Change of Base** and then solve (put in calculator!)

*If you do not have a base 10 log, you must use:

the Change of base formula — or — your calculator:

Chase of Base Formula: $\log_b M =$ ____________

ALPHA, WINDOW, #5 logBase(

1. $\log_2 9 =$

2. $\log_5 510 =$

Properties of Logs

Product Property: $\log_b MN = \log_b M + \log_b N$

Quotient Property: $\log_b \frac{M}{N} = \log_b M - \log_b N$

Power Property: $\log_b M^x = x \log_b M$

Examples: Rewrite each expression as a single log:

1. $\log 2 + \log 3 =$

2. $\log_4 64 - \log_4 16 =$

3. $6 \log_5 x + \log_5 y$

4. $\log 8 - 2 \log 6 + \log 3$

Expand each logarithm:

5. $\log (x^3 y^5)$

6. $\log_7 (22 xyz)$

7. $\log_6 [7 (2x – 3)]$

8. $\log_3 (x \sqrt{7})$

LOGARITHM LAWS
COMMON CORE ALGEBRA II

Logarithms have properties, just as exponents do, that are important to learn because they allow us to solve a variety of problems where logarithms are involved. Keep in mind that since logarithms give exponents, the laws that govern them should be similar to those that govern exponents. Below is a summary of these laws.

EXPONENT AND LOGARITHM LAWS

LAW	EXPONENT VERSION	LOGARITHM VERSION
Product	$b^x \cdot b^y = b^{x+y}$	$\log_b(x \cdot y) = \log_b x + \log_b y$
Quotient	$\dfrac{b^x}{b^y} = b^{x-y}$	$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$
Power	$\left(b^x\right)^y = b^{x \cdot y}$	$\log_b\left(x^y\right) = y \cdot \log_b x$

***Exercise* #1:** Which of the following is equal to $\log_3(9x)$?

(1) $\log_3 2 + \log_3 x$ (3) $2 + \log_3 x$

(2) $2\log_3 x$ (4) $x + \log_3 2$

***Exercise* #2:** The expression $\log\left(x^2/1000\right)$ can be written in equivalent form as

(1) $2\log x - 3$ (3) $2\log x - 6$

(2) $\log 2x - 3$ (4) $\log 2x - 6$

***Exercise* #3:** If $a = \log 3$ and $b = \log 2$ then which of the following correctly expresses the value of $\log 12$ in terms of a and b?

(1) $a^2 + b$ (3) $2a + b$

(2) $a + b^2$ (4) $a + 2b$

***Exercise* #4:** Which of the following is equivalent to $\log_2\left(\sqrt{x}/y^5\right)$?

(1) $\sqrt{\log_2 x} - 5\log_2 y$ (3) $\dfrac{1}{2}\log_2 x - 5\log_2 y$

(2) $2\log_2 x + 5\log_2 y$ (4) $2\log_2 x - 5\log_2 y$

***Exercise* #5:** The value of $\log_3\left(\sqrt{5}/27\right)$ is equal to

(1) $\frac{\log_3 5-6}{2}$ (3) $\frac{\log_3 5-3}{2}$

(2) $2\log_3 5+3$ (4) $2\log_3 5-3$ ________

***Exercise* #6:** If $f(x)=\log(x)$ and $g(x)=100x^3$ then $f(g(x))=$

(1) $100\log x$ (3) $300\log x$

(2) $6+\log x$ (4) $2+3\log x$

____

***Exercise* #7:** The logarithmic expression $\log_2\sqrt{32x^7}$ can be rewritten as

(1) $\sqrt{\log_2 35x}$ (3) $\sqrt{5+7\log_2 x}$

(2) $\frac{5+7\log_2 x}{2}$ (4) $\frac{35+\log_2 x}{2}$

____

***Exercise* #8:** If $\log 7=k$ then $\log(4900)$ can be written in terms of k as

(1) $2(k+1)$ (3) $2(k-3)$

(2) $2k-1$ (4) $2k+1$ ________

The logarithm laws are important for future study in mathematics and science. Being fluent with them is essential. Arguably, the most important of the three laws is the power law. In the next exercise, we will examine it more closely.

***Exercise* #9:** Consider the expression $\log_2\left(8^x\right)$.

(a) Using the third logarithm law (the Product Law), rewrite this as equivalent product and simplify.

(b) Test the equivalency of these two expressions for $x=0,1,$ and 2.

(c) Show that $\log_2\left(8^x\right)=3x$ by rewriting 8 as 2^3.

SOLVING EXPONENTIAL EQUATIONS USING LOGARITHMS

There are three different types of log equations:

A) **Equations with a log on one side only**:

Rule: "Do the Loop!"

1. $2 \log_4 n = 10$

2. $\log_{64} y = ½$

You Try These!

a. $\text{Log}_{64} x = 1/3$

b. $\log_3 x = 3$

B) **Equations with logs (same base) on both sides** :

Rule: Drop the logs!

3. $\log_5 (3x - 1) = \log_5 (2x + 7)$

4. $\log (p^2 - 2) = \log p$

You try these!

c. $\log (x^2 + 1) = 1$

d. $\log_7 (3y + 4) = \log_7 (6y - 20)$

III. **Equations with <u>no</u> logs** (using the Change of Base Formula):

Example: Solve each equation:

Do the Loop!

5. $5^{2x} + 4 = 20$

6. $3^{x+4} = 101$

You try these!

e. $2^{x-3} + 6 = 20$

f. $4^{3x} = 4096$

Practice: Solve for x. (Determine which method to use!)

1. $3^{x-1} = 24$

2. $2 \log x = -4$

3. $\log (x^2 + 6x) = \log (55)$

3. $5^{3x} - 16 = 484$

MENTAL MATH!

1. $2^x = \frac{1}{2}$

2. $3^x = 27$

3. $\log_9 3 = x$

4. $\log_4 64 = x$

5. $\log_8 2 = x$

6. $10^x = 1/100$

7. $\text{Log}_5 125 = x$

8. $25^x = 1/5$

THE NUMBER **e** AND THE NATURAL LOGARITHM
COMMON CORE ALGEBRA II

There are many numbers in mathematics that are more important than others because they find so many uses in either mathematics or science. Good examples of important numbers are 0, 1, i, and π. In this lesson you will be introduced to an important number given the letter **e** for its "inventor" Leonhard Euler (1707-1783). This number plays a crucial role in Calculus and more generally in modeling exponential phenomena.

The number "e" can be used as a base for exponents. (REMEMBER: $A = Pe^{rt}$)

The function $y = e^x$ has an inverse, the **natural logarithmic function**.

If $y = e^x$, then $\log_e y =$ ____. This is commonly written as $\ln y = x$.

The properties of common logarithms apply to natural logarithms

Example 1: Write as a single logarithm:

1. $2 \ln 12 - \ln 9 =$ _______________

2. $\frac{1}{3}(\ln x + \ln y) - 4 \ln z =$ _______________

Example 2: Simplify. (Hint: you may use your calculator!)

3. $\ln 1 =$ ________

4. $10 \ln e =$ _______

5. $\frac{\ln e^2}{2} =$ _______

<u>Solving Natural Log equations – 2 Methods</u>!

Type 1) Solving equations with "ln" on one side: "Do the Loop!" (Hint: ln = $\log_e$)

a) $\ln (2x - 4)^3 = 6$

b) $\ln (x + 2) - \ln 4 = 3$

Type 2) Solving an exponential equation with "e". (Hint: natural log each side!)

a) $e^{x/9} - 8 = 6$

b) $4e^{3x} + 1.2 = 14$

You Try!

1. $\ln (x + 6)^2 = 18$

2. $\ln (2x - 5) + \ln 3 = 27$

3. $2e^x - 4 = 16$

4. $e^{x+9} + 1 = 49$

Word Problems: Solve these with an algebraic approach!

1. An initial investment of $200 is now valued at $254.25. The interest rate is 6%, compounded continuously. How long has the money been invested?

2. A laboratory assistant tests an antibiotic that causes a rate of decline of 0.14. How long should it take for a population of 8000 to shrink to 500?

Word Problem Practice!

Write an equation and solve algebraically.

1. There are initially 1000 bacteria in a culture. The number of bacteria doubles each hour. The number of bacteria N present after t hours is $N = 1000(2)^t$. How long will it take the culture to increase to 50,000 bacteria?

2. If Mike deposits $1000 in an account paying 3.4% annual interest compounded continuously, how long will it take Mike's account to double?

3. A furniture store is closing out its business. Each week the owner lowers prices by 25%. After how many weeks will the sale price of a $500 item drop below $100?

4. As technology advances, the price of many graphing calculators goes down. If your graphing calculator cost $104 and the price decreases each year by 9%, after how many years will your calculator be worth half its original value?