ALGEBRA & FUNCTIONS

FACTORING

The Order of Factoring:

Greatest Common Factor (GCF)
↓
Difference of Two Perfect Squares (DOTS)
↓
Trinomial (TRI)
↓
"AC" Method / Earmuff Method (AC)
↓
Quadratic Formula (QF)

GCF:

$$ab + ac = a(b + c)$$

DOTS:

$$x^2 - y^2 = (x + y)(x - y)$$

TRI:

$$x^2 - x + 6 = (x + 2)(x - 3)$$

AC (a≠1):

$$2x^2 + 15x + 18$$

$$x^2 + 15x + 36$$

$$(x + 12)(x + 3)$$

$$(x + \tfrac{12}{2})(x + \tfrac{3}{2})$$

$$\boxed{(x + 6)(2x + 3)}$$

QF:

If all else fails to find the roots to a quadratic, use the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

DIVIDING POLYNOMIALS

Division Algorithm: $\dfrac{Dividend}{Divisor} = Quotient + \dfrac{Remainder}{Divisor}$

Long Division of Polynomials:

$$(2x^2 + 7x + 6) \div (x + 2)$$

$x + 2 \,\overline{)\, 2x^2 + 7x + 6}$, quotient $2x + 3$

multiply

2x was needed to create the first term of $2x^2$

$2x^2 + 4x$ ← (subtract)

$3x + 6$

$3x + 6$ ← (subtract)

0 remainder

$$\boxed{2x + 3}$$

Synthetic Division of Polynomials:

$$(x^3 + 6x^2 + 7x - 6) \div (x + 4)$$

$x + 4 = 0$
$\;\;\;-4\;\;-4$
$\boxed{x = -4}$

-4	1	6	7	-6
	↓	-4	-8	4
	1	2	-1	$\boxed{-2}$

multiply; Add; Add; Add; Remainder

$$\boxed{x^2 + 2x - 1 + \frac{-2}{x+4}}$$

OTHER FORMS OF FACTORING

Factor by Grouping:

$$x^3 + 2x^2 - 3x - 6$$

$$x^2(x + 2) - 3(x + 2)$$

$$\boxed{(x^2 - 3)(x + 2)}$$

Factoring Perfect Cubes by SOAP:

S – "Same" as the sign in the middle of the original expression"

O – "Opposite" sign

AP – "Always Positive"

$$x^3 - 8$$

$$(x)^3 - (2)^3$$

$$\boxed{(x - 2)(x^2 + 2x + 4)}$$

Perfect Cube Factor   SOAP Factor

THE REMAINDER THEOREM

When the polynomial $f(x)$ is divided by a binomial in the form of $(x - a)$, the remainder equals $f(a)$.

$$\frac{4x^2 + 2x - 5}{(x - 1)}$$

$$f(1) = 4(1)^2 + 2(1) - 5 \Rightarrow \boxed{1}$$

The remainder is 1!

THE FACTOR THEOREM

If $f(a) = 0$ for polynomial $f(x)$, then a binomial in the form of $(x - a)$ must be a factor of the polynomial.

$$\frac{x^4 + 6x^3 + 7x^2 - 6x - 8}{(x + 4)}$$

$$f(-4) = (-4)^4 + 6(-4)^3 + 7(-4)^2 - 6(-4) - 8$$
$$f(-4) = 256 + (-384) + 112 - (-24) - 8$$
$$f(-4) = 0$$

The remainder is zero, therefore $(x + 4)$ is a factor!

QUADRATIC: A *quadratic equation* is a polynomial equation with a degree of two (2).

THE STANDARD FORM OF A QUADRTATIC EQUATION

The standard form of a quadratic is in the form of

$$ax^2 + bx + c = 0,$$

where a, b, and c are constants where a ≠ 0.

THE SUM OF THE ROOTS OF A QUADRATIC

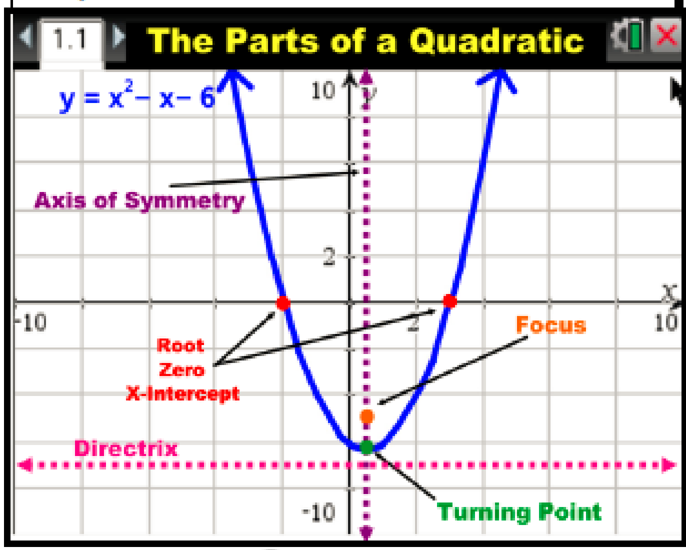
Sum of the Roots: $r_1 + r_2 = \frac{-b}{a}$

where a and b are constants from a quadratic equation in the form of $ax^2 + bx + c = 0$.

THE PRODUCT OF THE ROOTS OF A QUADRATIC

Product of the Roots: $r_1 \cdot r_2 = \frac{c}{a}$

where a and c are constants from a quadratic equation in the form of $ax^2 + bx + c = 0$.



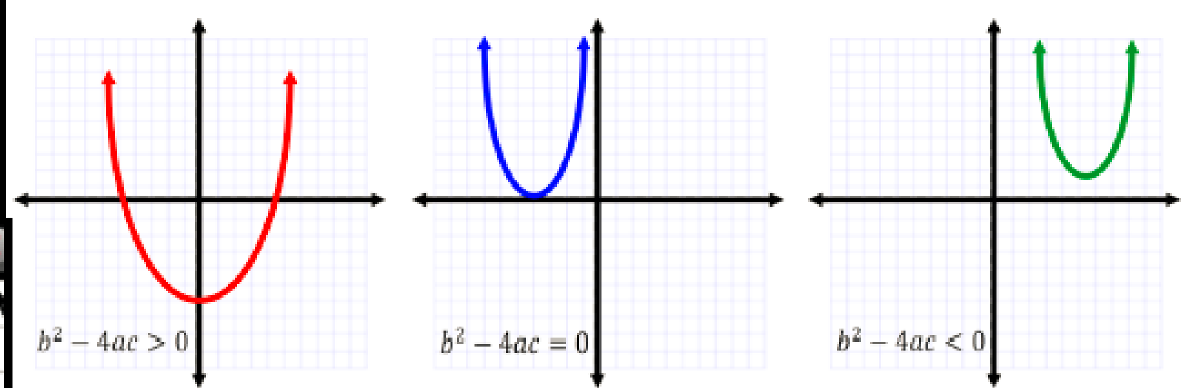
THE DISCRIMINANT

The discriminant is a part of the quadratic formula which allows mathematicians to anticipate the nature, or kinds of roots a particular quadratic equation will have.

$$b^2 - 4ac$$

where a, b, and c are constants

The Value of the Discriminant	The Nature of the Roots	Number of X-Intercepts
$b^2 - 4ac > 0$, and is a perfect square	Real, Rational, & Unequal	2
$b^2 - 4ac > 0$, and is *not* a perfect square	Real, Irrational, & Unequal	2
$b^2 - 4ac < 0$	Imaginary	0 (never touches the x-axis)
$b^2 - 4ac = 0$	Real, Rational, & Equal	1 (multiplicity of 2, called a *bounce*)



THE PARTS OF A QUADRATIC

Root/Zero/X-Intercept: a point on a quadratic where $f(x) = 0$. It is a point where the quadratic intersects the $x - axis$.

Turning Point(Vertex): the point on a quadratic where the direction of the function changes.

Axis of Symmetry: a line of symmetry in the form of $x = c$, where c is a constant. The value of c is the *same value* as the x value of the turning point.

Focus: a point which lies "inside" the parabola on the axis of symmetry.

Directrix: a line that is perpendicular to the axis of symmetry & lies "outside" the parabola.

FUNCTION: A function is a relation that consists of a set of ordered pairs in which each value of x is connected to a unique value of y based on the rule of the function. For each x value, there is one and only one corresponding value of y. A function also passes the vertical line test.

DOMAIN: The largest set of elements available for the independent variable, the first member of the ordered pair (x).

RESTRICTIONS ON DOMAIN:

1. **Fraction:** The denominator cannot be zero. Set the entire denominator equal to zero and solve.

$$f(x) = \frac{x-4}{x+3};\ \ x \neq -3$$

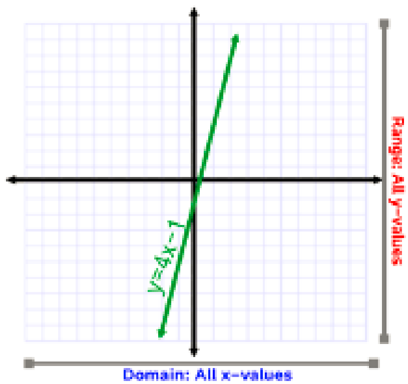
2. **Radical:** The radicand cannot be negative. Set the radicand greater than or equal to zero and solve.

$$f(x) = \sqrt{x-5};\ \ x \geq 5$$

3. **Radical in the Denominator:** The radical cannot be negative *and* the denominator cannot be zero. Set the radicand greater than zero and solve.

$$f(x) = \frac{1}{\sqrt{x+7}};\ \ x > -7$$

RANGE: The set of elements for the dependent variable, the second member of the ordered pair (y).



COMPOSITION FUNCTIONS: One function is substituted into another in place of the variable. This can involve numeric substitutions or substitutions of an algebraic expression in the function in the place of the variable.

NOTATION: $f(g(x))$ or $f \circ g(x)$

Always read from right to left when using this notation.

Example 1: If $f(x) = x + 9$ and $g(x) = 2x + 3$, find $f(g(3))$

$$g(3) = 2(3) + 3 \Rightarrow 6 + 3 = 9$$

$$f(9) = (9) + 9 = \boxed{18}$$

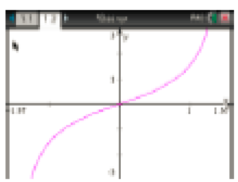
Example 2: If $f(x) = x + 5$ and $g(x) = 3x + 4$, find $f \circ g(x)$

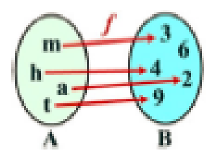
$$g(x) = x + 5$$

$$f(x+5) = 3(x+5) + 4 \Rightarrow 3x + 15 + 4 = \boxed{3x + 19}$$

ONE-TO-ONE FUNCTION

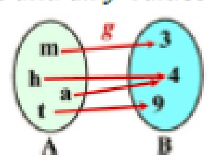
A one-to-one function must be a function, where when the ordered pairs are examined, the are no repeating x values or y values. One-to-one functions also pass *both* the horizontal and vertical line tests.





ONTO FUNCTION

All x values and all y values are used.



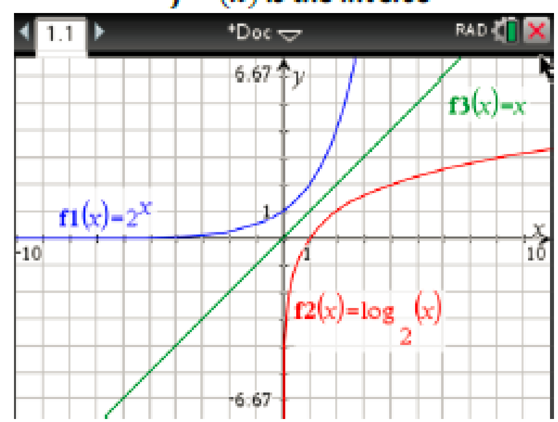
INVERSE FUNCTIONS:

The inverse of a function is the reflection of the function over the line $y = x$. Only a one-to-one function has an inverse function.

NOTATION:

$f(x)$ is the function

$f^{-1}(x)$ is the inverse



END BEHAVIOR

The *end behavior* of a graph is defined as what direction the function is heading at the ends of the graph. The end behavior can be determined by the following:

1. The degree of the function
2. The leading coefficient of the function

NOTATION:

As $x \to \pm\infty$, $f(x) \to \pm\infty$

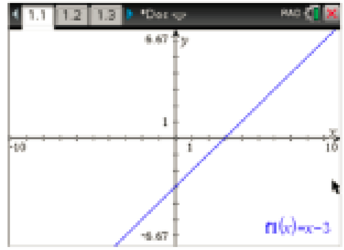
This notation is read as. "As x approaches positive/negative infinity, y approaches positive/negative infinity."

(*NOTE*: In Algebra 2, these are the only two notations you should know)

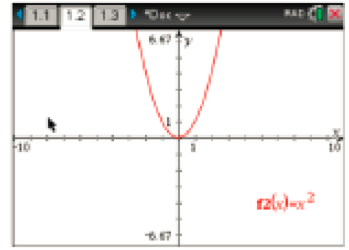
MULTIPLICITY

Multiplicity is defined as how many times a particular number is a zero for a given polynomial. In other words, it's the amount of times a root repeats itself given the features of the function.

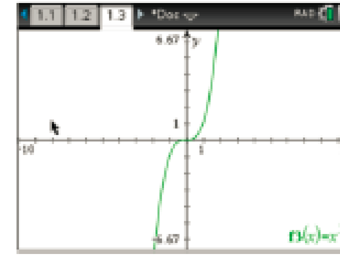
Multiplicity of 1 "Cut Thru"



Multiplicity of 2 "Bounce"



Multiplicity of 3 "Snake Thru"



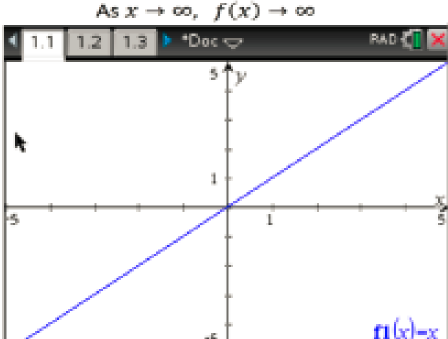
Odd Degree Polynomials

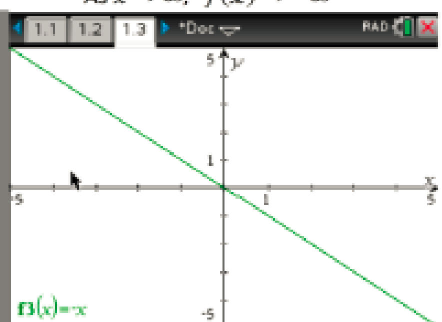
Positive Leading Coefficient

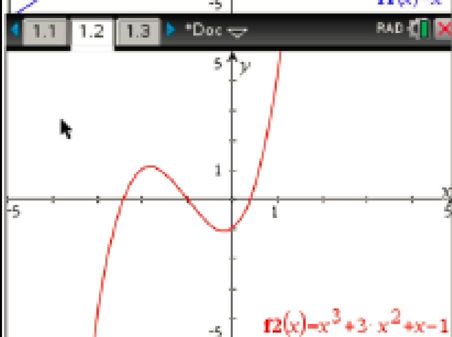
As $x \to -\infty$, $f(x) \to -\infty$
As $x \to \infty$, $f(x) \to \infty$

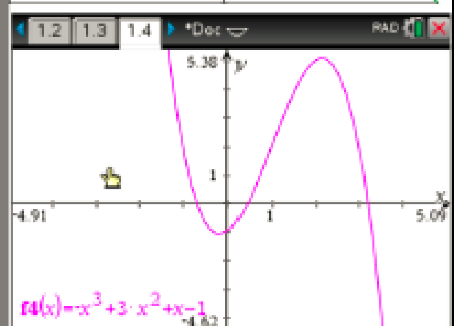
Negative Leading Coefficient

As $x \to -\infty$, $f(x) \to \infty$
As $x \to \infty$, $f(x) \to -\infty$









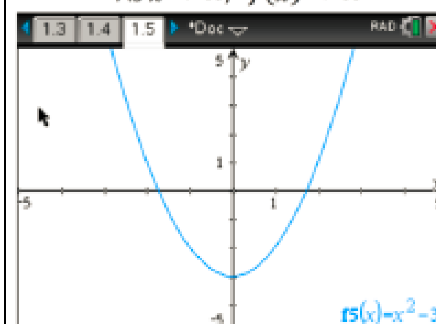
Even Degree Polynomials

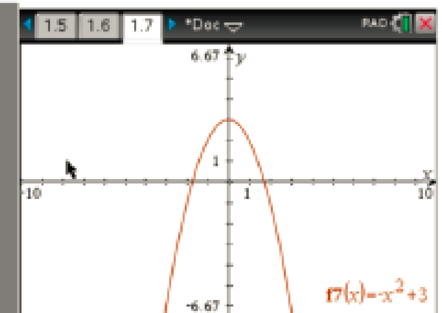
Positive Leading Coefficient

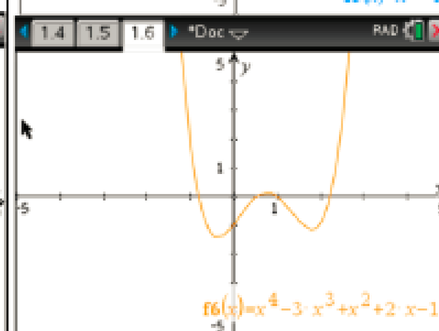
As $x \to -\infty$, $f(x) \to \infty$
As $x \to \infty$, $f(x) \to \infty$

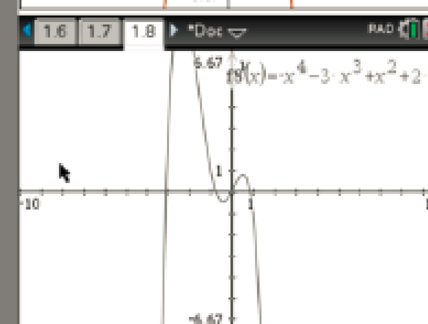
Negative Leading Coefficient

As $x \to -\infty$, $f(x) \to -\infty$
As $x \to \infty$, $f(x) \to -\infty$







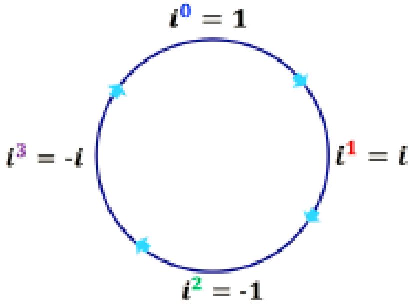


COMPLEX NUMBERS

The imaginary unit, i, is the number whose square is negative one.

$$\sqrt{-1} = i \quad \Leftrightarrow \quad i^2 = -1$$

The i-Clock



To solve for a value of i, you can use your calculator or you can use the i –clock!

Example: Solve for i^7

To solve, start at the top (i^0) and count around the clock at each quarter interval, and stop when you reach i^7. The answer is – i.

PROPERTIES OF EXPONENTS & RADICALS

$x^0 = 1$

$x^m \cdot x^n = x^{m+n}$

$x^{-m} = \dfrac{1}{x^m}$

$(x^n)^m = x^{n \cdot m}$

$\dfrac{x^m}{x^n} = x^{m-n}$

$\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$

$(xy)^n = x^n \cdot y^n$

$x^{\frac{p}{r}} = \sqrt[r]{x^p}$

$\sqrt[a]{x} = x^{\frac{1}{a}}$

$\sqrt[n]{a^n} = a$

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

RATIONAL EXPRESSIONS & EQUATIONS

To add or subtract rational expressions, you need to find a *common denominator!*

$$\frac{10}{2x^2} + \frac{5}{3x} \Rightarrow \frac{3}{3} \cdot \frac{10}{2x^2} + \frac{5}{3x} \cdot \frac{2x}{2x} \Rightarrow \frac{30}{6x^2} + \frac{10x}{6x^2} = \boxed{\frac{30 + 10x}{6x^2}}$$

To multiply rational expressions, factor first, reduce, and then multiply through.

$$\frac{6a}{3a + 15} \cdot \frac{4a + 20}{2a^2} \Rightarrow \frac{\overset{2}{\cancel{6a}}}{\underset{1}{\cancel{3}}\cancel{(a+5)}} \cdot \frac{\overset{2}{\cancel{4}}\cancel{(a+5)}}{\cancel{2a^2}_{a}} \Rightarrow \frac{2}{1} \cdot \frac{2}{a} = \boxed{\frac{4}{a}}$$

To divide rational expressions, flip the second fraction, factor, reduce, and then multiply through.

$$\frac{6x + 18}{4} \div \frac{x^2 + 3x}{5x^2} \Rightarrow \frac{6x + 18}{4} \cdot \frac{5x^2}{x^2 + 3x} \Rightarrow \frac{\overset{3}{\cancel{6}}\cancel{(x+3)}}{\underset{2}{\cancel{4}}} \cdot \frac{5\cancel{x^2}^{x}}{\cancel{x}\cancel{(x+3)}} = \boxed{\frac{15x}{2}}$$

Complex Fractions:
Multiply each fraction by the LCD, cancel what's common. & simplify.

$$\frac{\frac{2}{x^2} - \frac{4}{x}}{\frac{4}{x} - \frac{2}{x^2}} \quad (\times x^2) \Rightarrow \frac{2 - 4x}{4x - 2} = \boxed{-1}$$

LOGARITHMS

$$B^e = N \quad \Leftrightarrow \quad \log_B N = e$$

Exponential Form — Logarithmic Form

An exponent and a logarithm are *inverses* of each other!

Properties of Logarithms

Do the loop

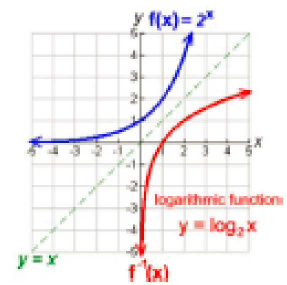
$\log_b(m \cdot n) = \log_b m + \log_b n$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b m^r = r \log_b m$

$\log_b b = 1$

$\log_b 1 = 0$



Properties of Natural Logarithms

Do the loop

$\ln(ab) = \ln a + \ln b$

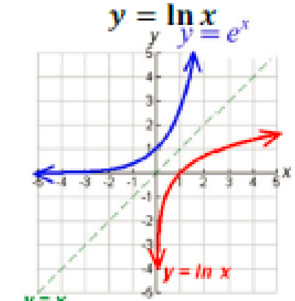
$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln a^b = b \ln a$

$\ln 1 = 0$

$\ln e = 1$

The inverse of $y = e^x$ is $y = \ln x$



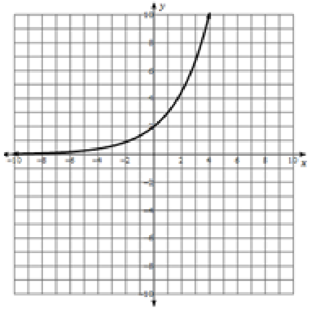
Exponential Graphs

ALL exponential word problems are based on the equation

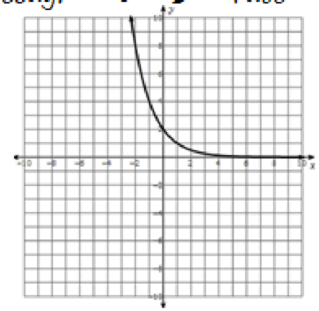
$$y = a(b)^x$$

- "a" is the starting value (it's also the y-intercept!)
- "b" is the rate of growth or decay

Growth: $b = 1 + \text{rate}$



Decay: $b = 1 - \text{rate}$



- "x" is the amount of time that has passed

Exponential Function Word Problems - % over time

- **Half-Life**: $y = a\left(\frac{1}{2}\right)^{\frac{\text{actual time elapsed}}{\text{half-life}}}$

- **Compound Interest** (compounded monthly, quarterly, etc.)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Continuously Compounded Interest (problem will ALWAYS say "continuously"!)

$$A = Pe^{rt}$$

A = Final Amount **P** = Principal **r** = rate **t** = time (yrs) **n** = # times compounded/year

Circles | **Circle Definition:** A 2-dimensional shape made by drawing a curve that is always the same distance from the center.

Circle Equations

General/Standard Equation of a Circle:

$$x^2 + y^2 + Cx + Dy + E = 0$$

where *C*, *D*, and *E* are constants.

Center – Radius Equation of a Circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

where *(h, k)* is the center and *r* is the radius.

Review of Factoring

The order of Factoring:

Greatest Common Factor (GCF)

↓

Difference of Two Perfect Squares (DOTS)

↓

Trinomial (TRI)

GCF:

$$ab + ac = a(b + c)$$

DOTS:

$$x^2 - y^2 = (x + y)(x - y)$$

TRI:

$$x^2 - x + 6 » (x + 2)(x - 3)$$

Completing the Square

The method of "completing the square" is used when factoring by the basic "Trinomial Method", or "AM" method cannot be applied to the problem. The completing the square method is commonly used in ***geometry to express a general circle equation in center-radius form.***

Example: Express the general equation $x^2 + 4x + y^2 - 6y - 12 = 0$ in center-radius form.

$$x^2 + 4x + y^2 - 6y - 12 = 0$$

$$x^2 + 4x + y^2 - 6y = 12$$

$$x^2 + 4x + \_\_ + y^2 - 6y + \_\_ = 12 + \_\_ + \_\_$$

$$x^2 + 4x + \mathbf{4} + y^2 - 6y + \mathbf{9} = 12 + \mathbf{4} + \mathbf{9}$$

$$(x + 2)(x + 2) + (y - 3)(y - 3) = 25$$

$$\boxed{(x + 2)^2 + (y - 3)^2 = 25}$$

Formula: $\left(\frac{b}{2}\right)^2$

Steps:

1) Determine if the squared terms have a coefficient of 1
2) If there is a constant/number on the left side of the equal sign, move that constant to the right side
3) Insert "boxes" or "blank spaces" after the linear terms to acquire a perfect-square trinomial
4) Take half of the linear term(s) and square the number. Insert this number on both the left and right sides
5) Factor using the "trinomial method"
6) Write your equation

Graphing Circles

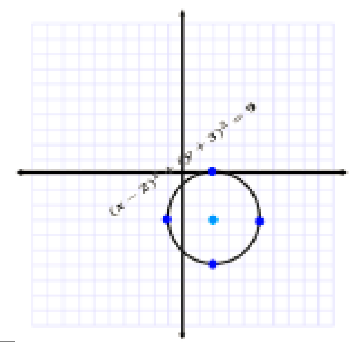
Steps:

1) Determine the center and the radius
2) Plot the center on the graph
3) Around the center, create four loci points that are equidistant from the center of the circle
4) Using a compass or steady freehand, connect all four points
5) Label when finished

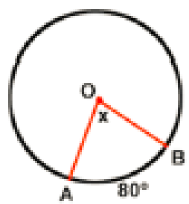
Example: Graph $(x - 2)^2 + (y + 3)^2 = 9$

The center is the point (2,-3)

The radius is 3

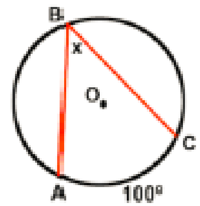


Angle Relationships in a Circle



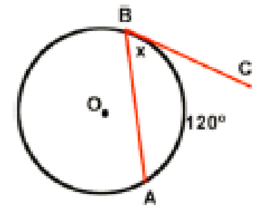
Central Angle:

$\measuredangle x = \widehat{AB}$



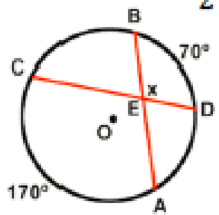
Inscribed Angle:

$\measuredangle x = \frac{1}{2}\widehat{AC}$



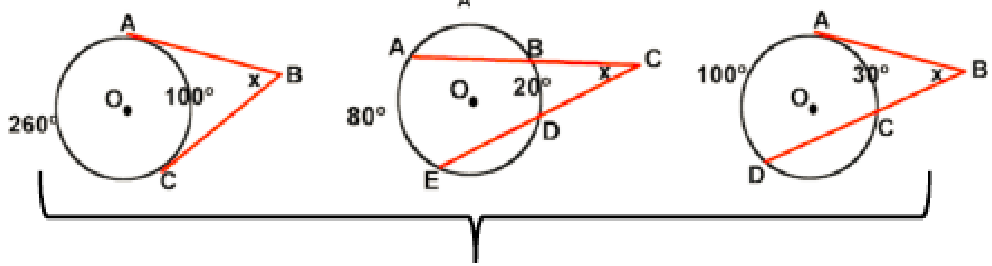
Tangent-Chord Angle:

$\measuredangle x = \frac{1}{2}\widehat{AC}$

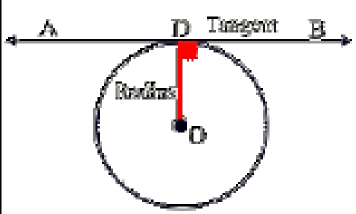


Two Chord Angles:

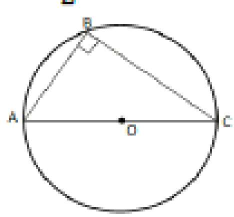
$\measuredangle x = \frac{\widehat{Arc_1} + \widehat{Arc_2}}{2}$



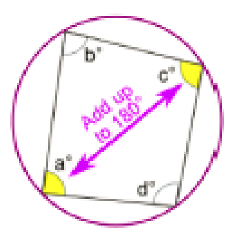
$\frac{\widehat{Big} - \widehat{Little}}{2} = \measuredangle x$



A tangent is perpendicular to its radius, forming a 90° angle

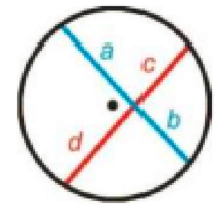


An angle that is inscribed in a semicircle equals 90°



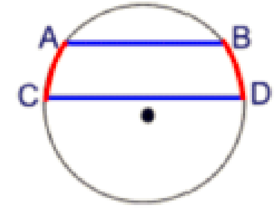
If a quadrilateral is inscribed in a circle, then its opposite angles = 180°

Segment Relationships in a Circle



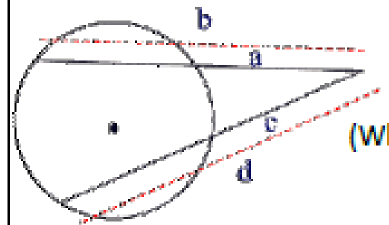
(Part)(Part)=(Part)(Part)

$(a)(b) = (c)(d)$



If $\overline{AB} \parallel \overline{CD}$, then $\widehat{AC} \cong \widehat{BD}$

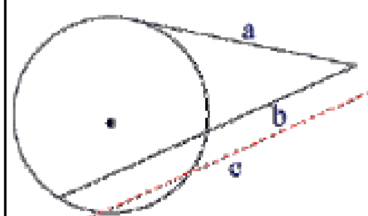
Parallel chords intercept congruent arcs



(W)(E) = (W)(E)

(Whole)(External)=(Whole)(External)

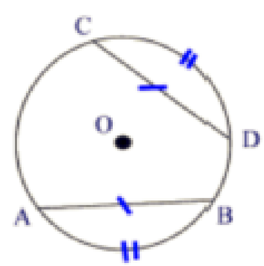
$(b)(a) = (d)(c)$



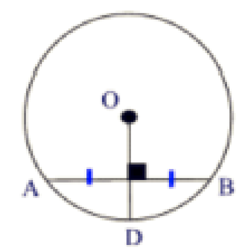
(W)(E) = $(T)^2$

(Whole)(External)=$(\text{Tangent})^2$

$(c)(b) = (a)^2$



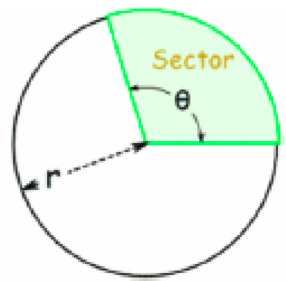
If $\overline{AB} \cong \overline{CD}$, then $\widehat{AB} \cong \widehat{CD}$



If a diameter/radius is perpendicular to a chord, then the diameter/radius bisects the chord and its arc.

Circles (Con't)

Area of a Sector



$$A = \frac{1}{2}r^2\theta$$

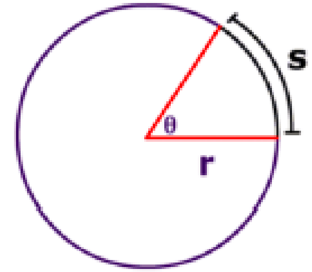
where A is the area of the sector, r is the radius, and θ is an angle in radians.

-or-

$$A = \frac{n}{360}\pi r^2$$

where A is the area of the sector, n is the amount of degrees in the central angle, and r is the radius

Sector Length

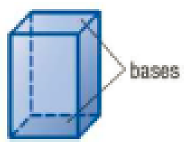


$$s = r\theta$$

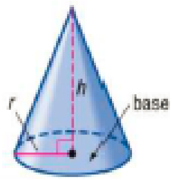
where s is the sector length, r is the radius, and θ is an angle in radians.

3D Figures

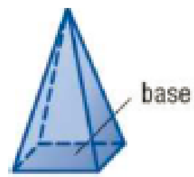
Prism



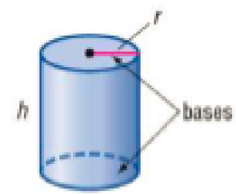
Cone



Pyramid



Cylinder



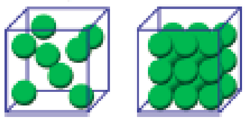
Sphere

Cavalieri's Principle: If two solids have the same height and the same cross-sectional area at every level. then the solids have the same volume.

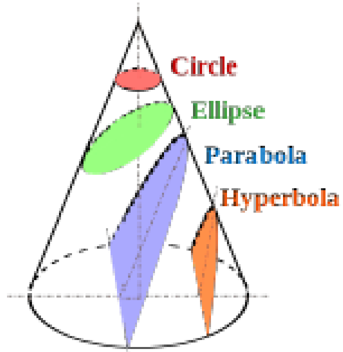
Density Formulas:

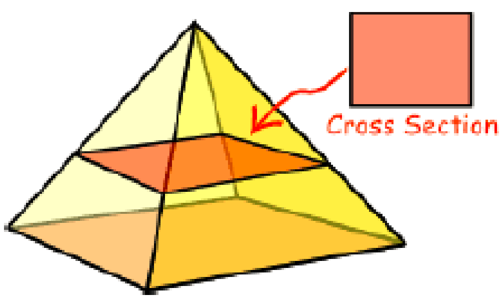
$$Mass = (Density) \cdot (Volume)$$

$$Density = \frac{(Mass)}{(Volume)}$$

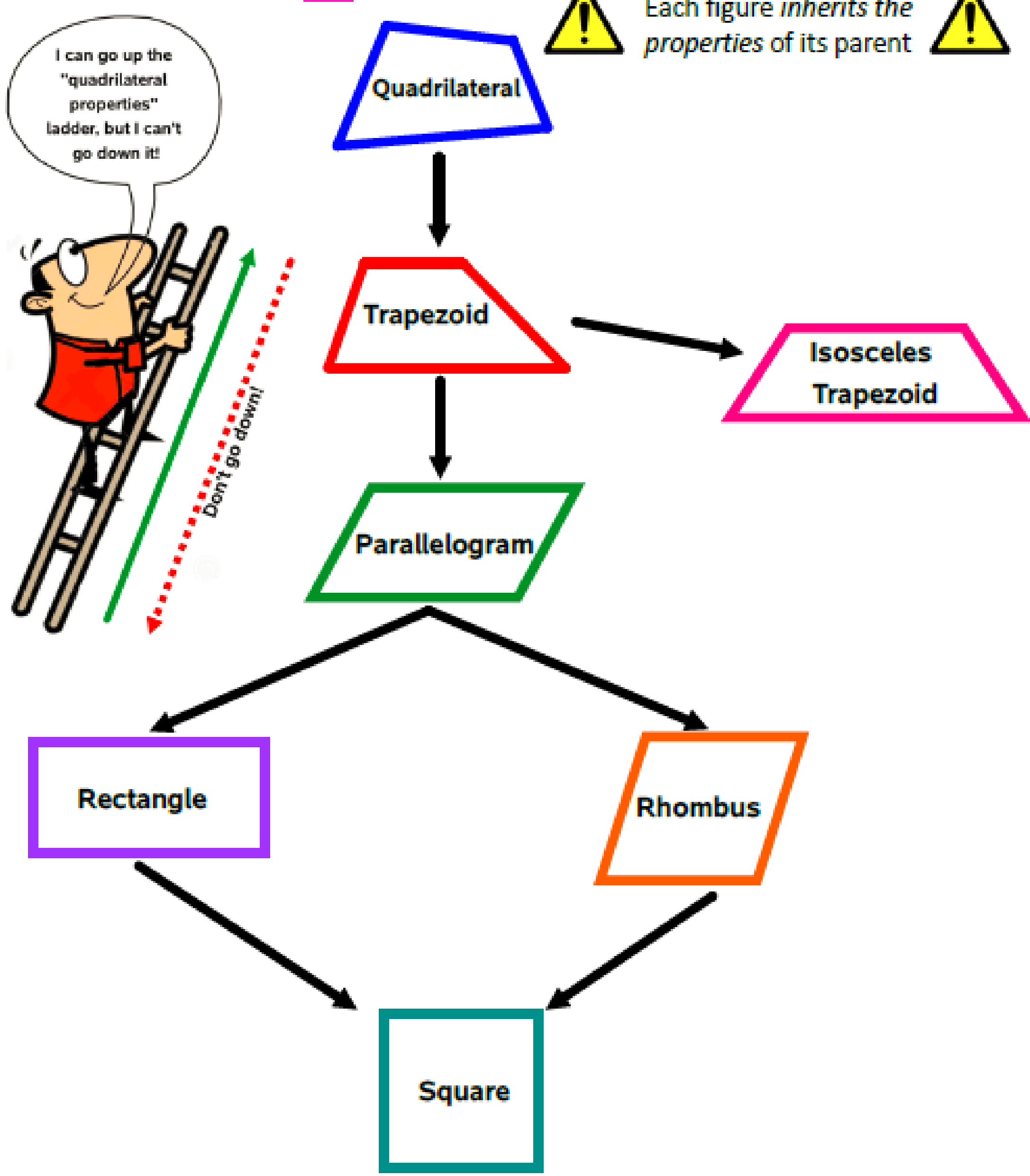


Cross Sections: a surface or shape that is or would be exposed by making a straight cut through something.





The Quadrilateral Family Tree



The Quadrilateral Properties

Quadrilateral

- ✓ A **quadrilateral** is a four-sided polygon

Trapezoid

- ✓ at least **one pair** of parallel sides

Formula: The length of the **median** of a trapezoid can be calculated using the following formula:

$$Median = \frac{1}{2}(Base_1 + Base_2)$$

Isosceles Trapezoid

- ✓ each pair of base angles are congruent
- ✓ diagonals are congruent
- ✓ one pair of congruent sides (which are the called the *legs*. These are the non-parallel sides)

Parallelogram

- ✓ opposite sides are parallel
- ✓ opposite sides are congruent
- ✓ opposite angles are congruent
- ✓ consecutive angles are supplementary
- ✓ diagonals bisect each other

Rectangle

- ✓ all angles at its vertices are right angles
- ✓ diagonals are congruent

Rhombus

- ✓ all sides are congruent
- ✓ diagonals are perpendicular
- ✓ diagonals bisect opposite angles
- ✓ forms four congruent right triangles
- ✓ forms two pairs of two congruent isosceles triangles

Square

- ✓ diagonals form four congruent isosceles right triangles

TRIGONOMETRY & TRIGONOMETRIC FUNCTIONS

TRIGONOMETRIC FUNCTIONS

$\sin\theta = \dfrac{opposite}{hypotenuse}$ $\tan\theta = \dfrac{opposite}{adjacent}$ $\sec\theta = \dfrac{hypotenuse}{adjacent}$

$\cos\theta = \dfrac{adjacent}{hypotenuse}$ $\csc\theta = \dfrac{hypotenuse}{opposite}$ $\cot\theta = \dfrac{adjacent}{opposite}$

RECIPROCAL FUNCTIONS

$\sin\theta = \dfrac{1}{\csc\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$ $\tan\theta = \dfrac{1}{\cot\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

RADIANS

To change from *degrees* to *radians*, multiply by $\frac{\pi}{180}$.

DEGREES

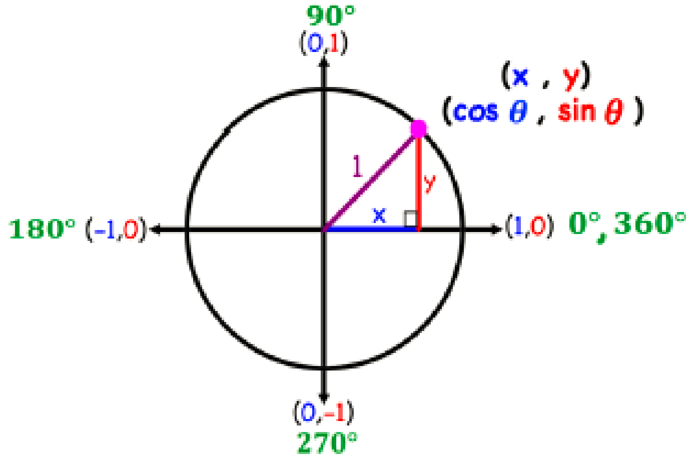
To change from *radians* to *degrees*, multiply by $\frac{180}{\pi}$.

ARC LENGTH OF A CIRCLE

$s = r \cdot \theta$

where s is the length of the sector, r is the length of the radius, and θ is an angle in radians

THE UNIT CIRCLE



THE UNIT CIRCLE – EXACT VALUES

Remember these facts & the table below!

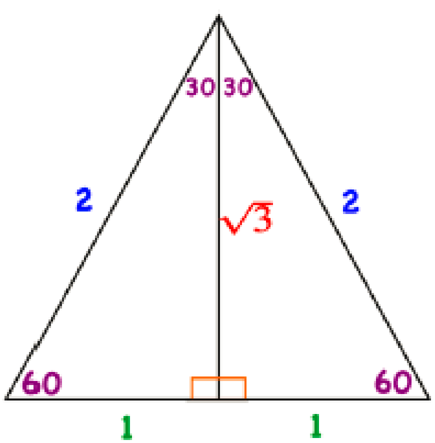
$cos\theta = x$

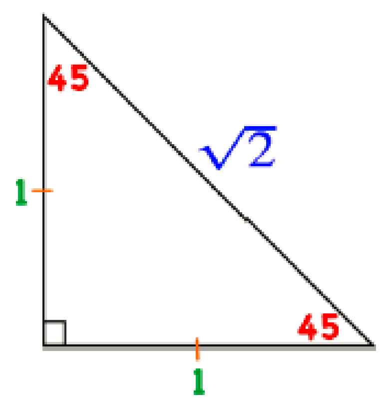
$sin\theta = y$

$tan\theta = \dfrac{y}{x} = \dfrac{sin\theta}{cos\theta}$

θ	0°	90°	180°	270°	360°
sin θ	0	1	0	-1	0
cos θ	1	0	-1	0	1
tan θ	0	UNDEF	0	UNDEF	0

SPECIAL RIGHT TRIANGLES



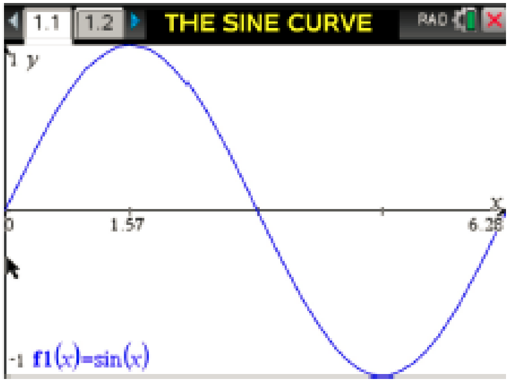


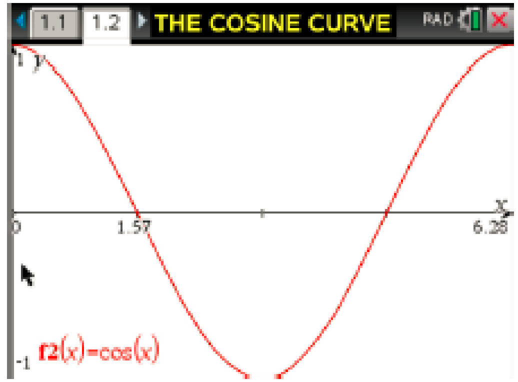
SPECIAL RIGHT TRIANGLES – EXACT VALUES

Remember the table below!

θ	30°	45°	60°
sin θ	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan θ	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

TRIGONOMETRIC GRAPHS





STANDARD FORMS OF TRIGONOMETRIC GRAPHS

$$y = A\sin(B(x-C)) + D \qquad y = A\cos(B(x-C)) + D$$

Amplitude (A): $\frac{1}{2}|Maximum - Minimum|$

Frequency (B): The number cycles the graph completes in 2π radians.

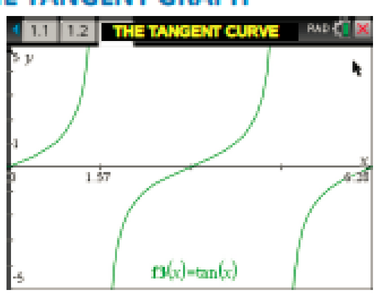
Horizontal Shift (C): The movement of a function left or right. The sign used in the equation is opposite the direction of the function.

Vertical Shift (D): The movement of a function up or down. The sign used in the equation is the same direction of the function.

Period: The horizontal length to complete one complete cycle. The formula to compute this horizontal distance is $\frac{2\pi}{b}$, where b is the frequency of the function.

Sketch Point: tells you where and how often to plot points. The formula is $\frac{Period}{4}$.

THE TANGENT GRAPH



Remember!

The tangent function is undefined at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ radians, which is why the function is discontinuous at those locations in the graph shown.

THE PYTHAGOREAN IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$

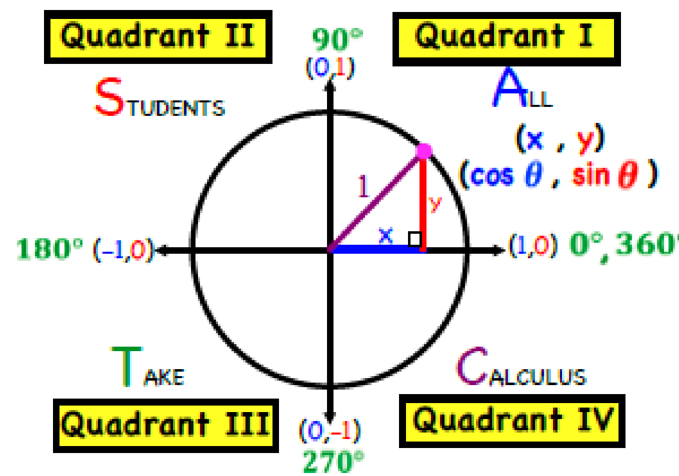
$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

INVERSE NOTATIONS

- The inverse of $y = \sin x$ is $y = \sin^{-1} x$ *or* $y = \arcsin(x)$
- The inverse of $y = \cos x$ is $y = \cos^{-1} x$ *or* $y = \arccos(x)$
- The inverse of $y = \tan x$ is $y = \tan^{-1} x$ *or* $y = \arctan(x)$

THE QUADRANTS & TRIGONOMETRIC RELATIONSHIPS



QUADRANT I: All trigonometric functions are positive

QUADRANT II: Only sine is positive

QUADRANT III: Only tangent is positive

QUADRANT IV: Only cosine is positive

STATISTICS & PROBABILITY

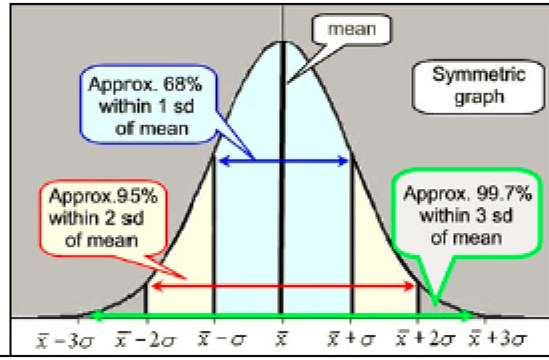
TYPES OF STATISTICAL STUDIES

Survey: used to gather large quantities of facts or opinions. Surveys are usually asked in the form of a question. For example, "Do you like Algebra, Geometry, or neither?" would be a survey question.

Observational Study: the observer does not have any interaction with the subjects and just examines the results of an activity. For example, the location as to where the Sun rises and sets on each day throughout the year.

Controlled Experiment: two groups are studied while an experiment is performed with one of them but not the other. For example, testing if orange juice has an effect in preventing the "common cold" with a group of 100 people, where 50 people will drink orange juice and the other 50 will not drink the juice. The statistician will then analyze the data of the control group and the experimental group.

THE NORMAL DISTRIBUTION CURVE



CONFIDENCE INTERVALS

A *Confidence interval* is a range or interval of values used to estimate the true value of a population parameter. The formula to calculate the confidence interval is given by:

$$C.I. = \bar{x} \pm z \cdot \frac{\sigma}{\sqrt{n}}$$

where σ is a known value, $\bar{x}$ is the mean, and z changes value depending on the confidence level.

Confidence Level	z
90 %	1.645
95 %	1.96
99 %	2.575

Z-SCORES

A z-score represents the number of standard deviations a given value x falls from the mean, μ.

Formula:

$$z = \frac{number - mean}{standard\ deviation} = \frac{x-\mu}{\sigma}$$

where x is the value being examined, μ is the population mean, and σ is the population standard deviation.

Notes:

- A negative z-score represents a value less than the mean.
- A z-score of zero represents the mean
- A positive z-score represents a value greater than the mean.