Name: ____________________________________ Date: __________________

2-1 QUADRATIC FUNCTION REVIEW

Linear and exponential functions are used throughout mathematics and science due to their simplicity and applicability. **Quadratic functions** comprise another very important category of functions. You studied these extensively in Math 1 and Math 2, but we will review many of their important characteristics in this unit.

QUADRATIC FUNCTIONS

Any function of the form $f(x) = ax^2 + bx + c$ where the leading coefficient, a, is not zero.

***Exercise* #1:** *Without* the use of your calculator, evaluate each of the following quadratic functions for the specified input values. Recall that, according to the formal Order of Operations, exponent evaluation should always come first.

(a) $f(x) = x^2$

$f(-3) =$

$f(5) =$

(b) $g(x) = 2x^2 - 5$

$g(2) =$

$g(-1) =$

(c) $h(x) = -x^2 + 4x$

$h(-2) =$

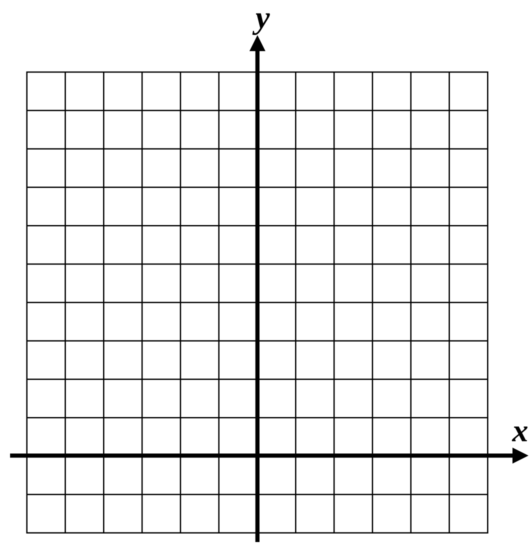
$h(3) =$

Graphs of quadratic functions form what are known as **parabolas**. The simplest quadratic function, and one that you should be very familiar with, is reviewed in the next exercise.

***Exercise* #2:** Consider the simplest of all quadratic functions $y = x^2$.

(a) Create a table of values to plot this function over the domain interval $-3 \le x \le 3$.

x	-3	-2	-1	0	1	2	3
$y = x^2$							



(b) Sketch a graph of this function on the grid to the right.

(c) State the coordinates of the **turning point** of this parabola.

(d) State the equation of this parabola's **axis of symmetry**.

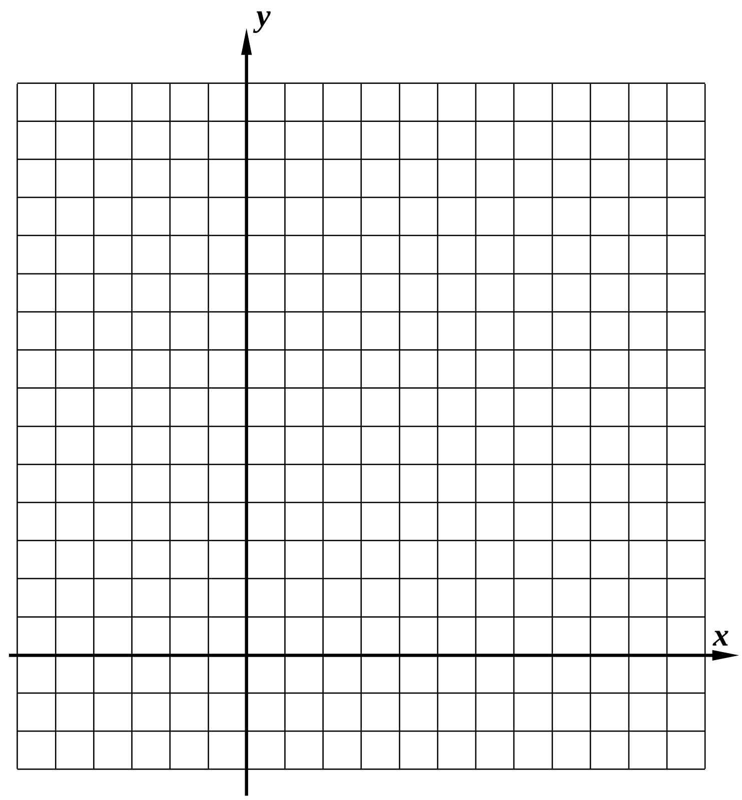
(e) Over what interval is this function increasing?

All quadratic functions that have unlimited domains (domains that consist of the set of all real numbers) have turning points and an axis of symmetry. It is important to be able to sketch a parabola using your graphing calculator to generate a table of values.

***Exercise* #3:** Consider the quadratic function $f(x) = -x^2 + 6x + 5$.

(a) Determine the turning point of this function.

(b) What is the range of this quadratic?

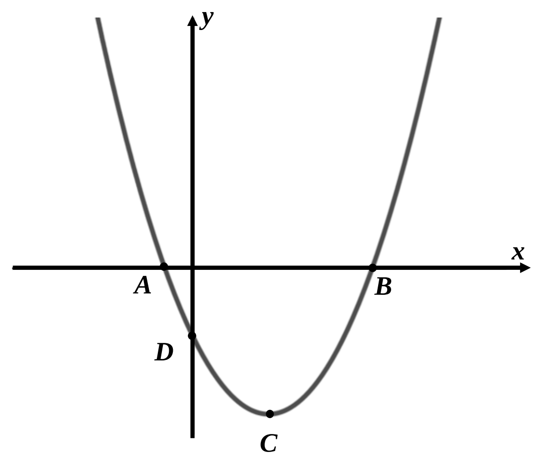


(c) Graph this function on the grid to the right.

(d) Why does this parabola open downward as opposed to $y = x^2$ which opened upward?

(e) Between what two consecutive integers does the larger solution to the equation $-x^2 + 6x + 5 = 0$ lie? Show this point on your graph.

***Exercise* #4:** A sketch of the quadratic function $y = x^2 - 11x - 26$ is shown below marked with points at its intercepts and its turning point. Using tables or a graph on your calculator, determine the coordinates for each of the points.



The x-intercepts: A B
(Zeroes)

The y-intercept: D

The turning point: C

Over what interval is this function positive?

2-2 MORE USEFUL FUNCTIONS
SQUARE ROOT – ABSOLUTE VALUE – CUBE ROOT

Square roots are operations on numbers that give exactly one output for a given input. So, they fit nicely into the definition of a function. We can graph the general square root function, once we establish a **very important fact about square roots**.

***Exercise* #1:** Consider $\sqrt{-4}$?

(a) Why are neither 2 nor -2 the correct square root of -4?

(b) What can you conclude about taking square roots of negative numbers? Explain

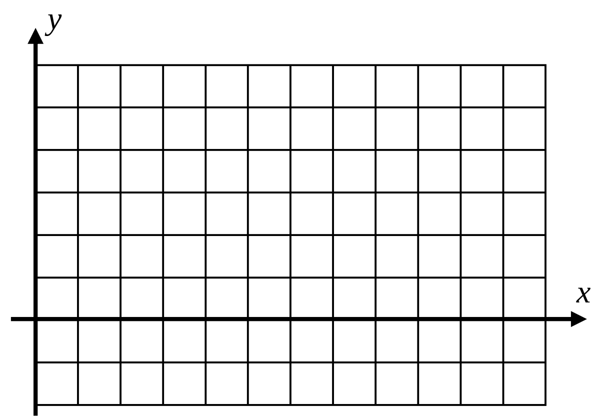
It is absolutely critical that you understand, deep down inside, why **finding the square root of a negative number** is **not possible** with any **real number**. Let's now get into the basic square root graph.

***Exercise* #2:** Consider $f(x)=\sqrt{x}$.

(a) Create a table of values for input values of x for which you can find rational square roots.

x				
$f(x)=\sqrt{x}$				

(b) Graph the function on the grid provided.



(c) What is the domain of this function?

(d) What is the range of this function?

(e) Circle the correct choice below that characterizes $f(x)=\sqrt{x}$.

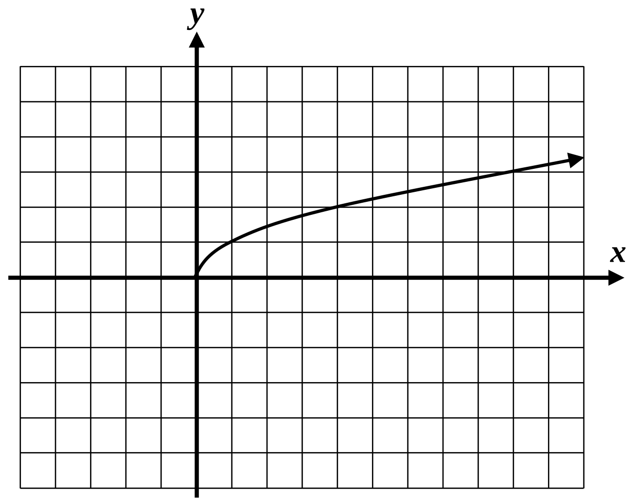
$f(x)$ is always decreasing

$f(x)$ is always increasing

(f) What shape does the square root graph appear to be "half" of? **This is not a coincidence.**

Square root graphs can be shifted just as quadratics can. And they shift in much the same way.

***Exercise* #3:** The graph of $y=\sqrt{x}$ is shown below.



(a) Using your calculator, graph the function given by $y=\sqrt{x+4}+2$. Show your table of values.

(b) State the domain and range of this function.

Domain:

Range:

(c) Graph the function given by $y=\sqrt{x-1}-4$. Show your table. Also state its domain and range.

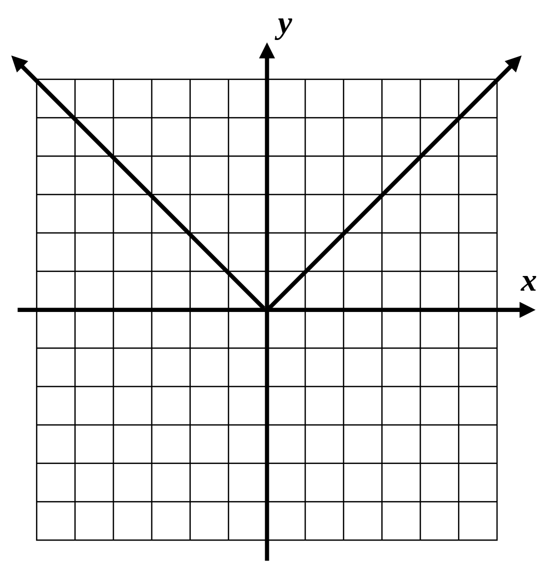
Table Domain: Range:

ABSOLUTE VALUE

***Exercise* #4:** The absolute value gives us the "size" or **magnitude** of a number. Find each of the following.

(a) $|-7|$ (b) $|-2|$ (c) $|6|$ (d) $|0|$

***Exercise* #5:** The graph of $y=|x|$ is shown on the grid below.



(a) Create a graph of $y=|x+3|-2$.

(b) State the domain and range of this function:

Domain: Range:

(c) Let's see if you get the pattern. Sketch $y=|x-2|-1$ without using your calculator.

Cube Roots

Just like square roots undo the squaring process, **cube roots**, undo the **process of cubing a number**. The cube root's technical definition along with its symbolism is given below.

Cube Roots

If $x^3 = a$ then $\sqrt[3]{a}$ is a solution to this equation. Or… $\sqrt[3]{a}$ is any number that when cubed gives a.

***Exercise* #1:** It is good to know some basic cube roots of smaller numbers. Find each of the following and justify by using a multiplication statement.

(a) $\sqrt[3]{8}$

(b) $\sqrt[3]{1}$

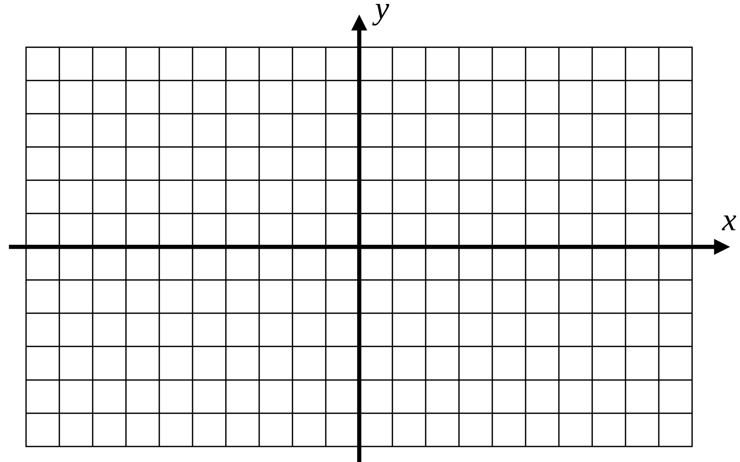
(c) $\sqrt[3]{27}$

***Exercise* #4:** Consider the basic cubic function $y = \sqrt[3]{x}$.

(a) Fill out the table of values below without the use of your calculator.

x	-8	-1	0	1	8
y					

(b) Plot its graph on the grid provided below.



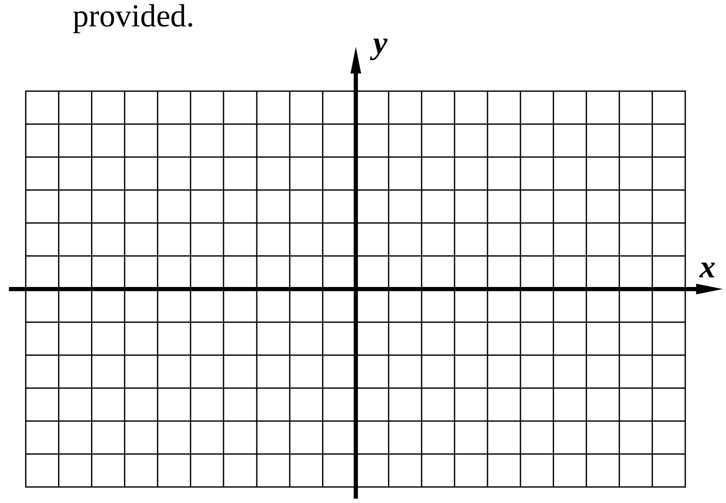
Just like with all other functions, cube root graphs can be **transformed** in a variety of ways. Let's see if our **shifting pattern** continues to hold with cube roots.

***Exercise* #5:** Consider the function $f(x) = \sqrt[3]{x+2} - 4$.

(a) Use your calculator to create a table of values that can be plotted. Show your table below.

x					
y					

(b) Create a graph of this function on the axes provided.

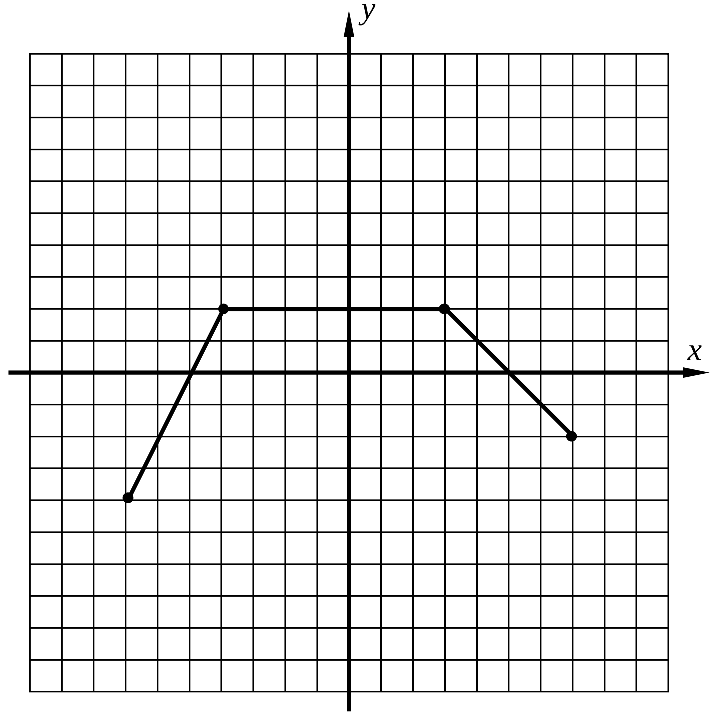


(c) Describe how the graph you drew in Exercise #4 was shifted to produce this graph?

2-3 FUNCTION TRANSFORMATIONS

We have transformed many functions this year by **shifting them** and **stretching them**. These transformations occur on a general basis and we will explore them in the next two lessons by looking almost exclusively at functions defined graphically. Still, we will rely heavily on function notation.

***Exercise* #1:** The function $y=f(x)$ is defined by the graph below. Answer questions based on this definition. Selected points are marked on the graph.



(a) Evaluate each of the following:

$f(3)=$ $\qquad\qquad$ $f(7)=$

$f(-4)=$ $\qquad\qquad$ $f(-7)=$

(b) State the zeroes of $f(x)$.

(c) Why is it impossible to evaluate $f(9)$?

(d) State the domain and range of $f(x)$.

Domain: $\qquad\qquad$ Range:

O.k. Now that we have a bit of a feel for $f(x)$ we are going to start to create other functions by **transforming** the function *f*.

***Exercise* #2:** Let's now define the function $g(x)$ by the formula $g(x)=2f(x)$.

(a) Evaluate each of the following. Show the work that leads to your answer. Remember, just follow the function's rule.

$g(-7)=$ $\qquad\qquad$ $g(-4)=$

$g(3)=$ $\qquad\qquad$ $g(7)=$

(b) How can you interpret the function rule in terms of the graph of $f(x)$?

(c) Sketch a graph of $f(x)$ on the grid above in Exercise #1. Write down points that you know are on $g(x)$ based on your answers to (a).

(d) State the domain and range of the function $g(x)$.

Domain: $\qquad\qquad$ Range:

So, we see from the last exercise that when a function gets multiplied by a constant, all of the *y*-values get multiplied by the same constant. This has the effect of "stretching" a function.

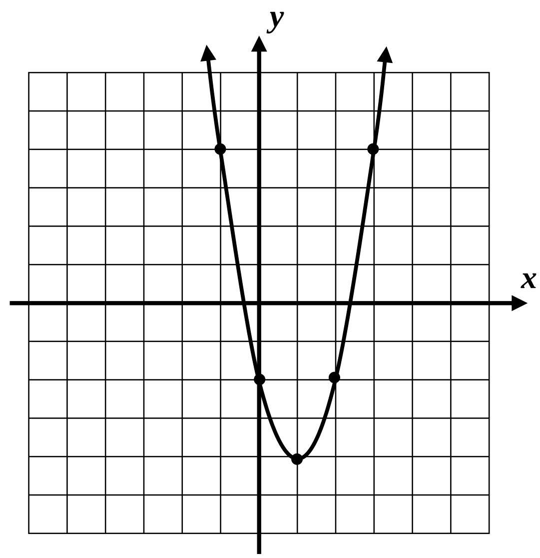
VERTICAL STRETCH

If the function $g(x)$ is defined by $g(x)=k\cdot f(x)$, then the graph of g will be stretched (or compressed) depending on the value of *k*. If *k* is negative, it will also **reflect** the function across the *x*-axis.

***Exercise* #3:** A quadratic $f(x)$ is shown below. The function $g(x)$ is defined by $g(x)=-\frac{1}{2}f(x)$.

(a) Calculate the values of $g(0)$ and $g(3)$. Show your work.

Explain the effect of multiplying by $-\frac{1}{2}$.



(b) Sketch an accurate graph of $g(x)$ on the same grid as $f(x)$.

(c) State the range of $g(x)$.

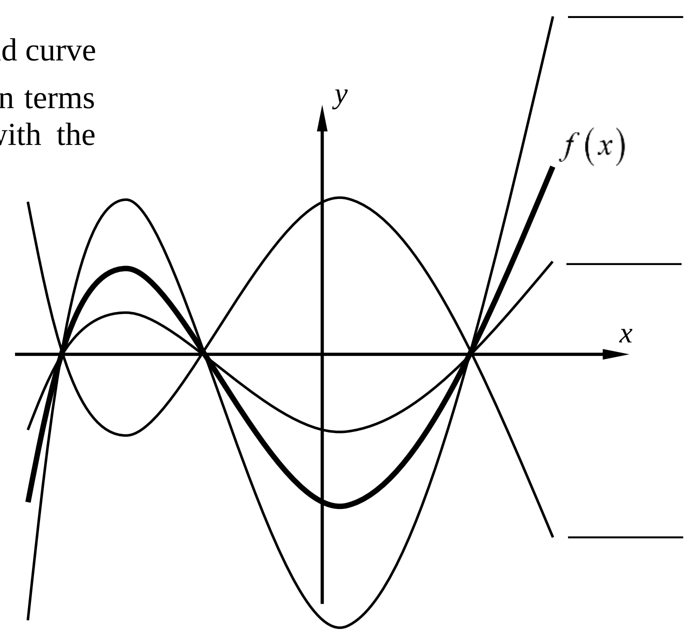
Let's do one final problem to see how well you understand what happens to the graph of a function when it has been multiplied by a constant.

***Exercise* #4:** The function $f(x)$ is graphed as the bold curve shown below. Three other functions are all defined in terms of f and are graphed as well. Label each curve with the appropriate function.

$g(x)=\frac{1}{2}f(x)$

$h(x)=-f(x)$

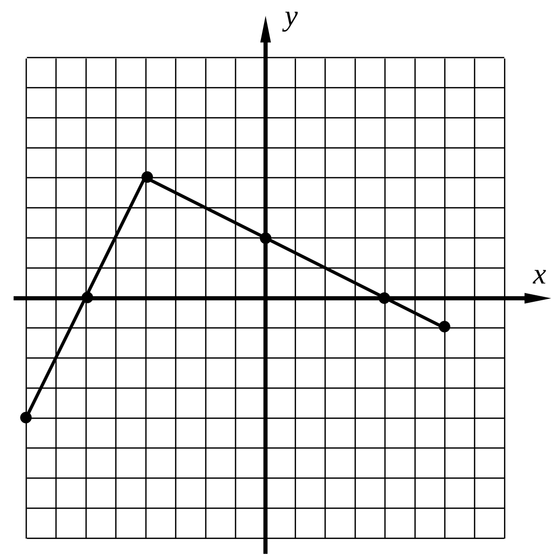
$k(x)=2f(x)$



2-4 HORIZONTAL STRETCHING OF FUNCTIONS

In the last lesson we saw how multiplying a function by a constant stretched (or compressed) the function's outputs, and thus its graph. This was a **vertical stretch** because it only affected the vertical (output) component of the function for a given input. In today's lesson, we will see what happens to a function when you first manipulate its input.

***Exercise* #1:** The function $f(x)$ is shown on the graph below. Selected points are shown as reference. The function $g(x)$ is defined by $g(x)=f(2x)$. Notice that the multiplication by 2 happens **before** f is even evaluated. This is tricky!



(a) Find the values of each of the following. Carefully follow the rule for $g(x)$ and show your work.

$g(2)=$ $\qquad$ $g(3)=$

$g(-2)=$ $\qquad$ $g(-4)=$

$g(0)=$ $\qquad$ $g(-3)=$

(b) Given the definition of $g(x)$, why can we **not** find a value for $g(4)$? Explain.

(c) State points that must lie on the graph of $g(x)$ based on your work in (a).

(d) Graph the function $g(x)$ based on your work from (b). Then, state the domain and range of both the original function, $f(x)$ and our new function $g(x)$. What remained the same? What changed?

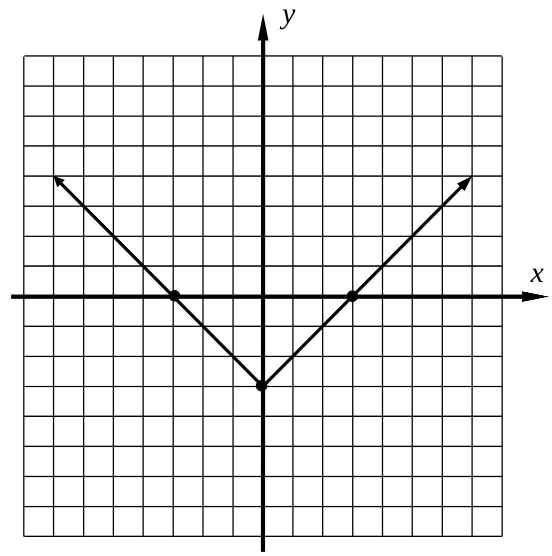
Original Function: $f(x)$ $\qquad$ **New Function:** $g(x)$

Domain: $\qquad$ Range: $\qquad$ Domain: $\qquad$ Range:

(e) Describe what happened to the graph of $f(x)$ when we multiplied the function's input by 2.

Notice how the **horizontal stretch** worked almost counter to what we would have thought. In other words, when we multiplied the *x*-value by 2, it **compressed** our graph by a factor of 2. The opposite would also occur.

***Exercise* #2:** The function $f(x)=|x|-3$ is shown on the graph below. The function $g(x)$ is defined by the formula $g(x)=\left|\frac{1}{2}x\right|-3$.



(a) Use your graphing calculator to produce a table of values for $g(x)$ and graph it on the grid to the right.

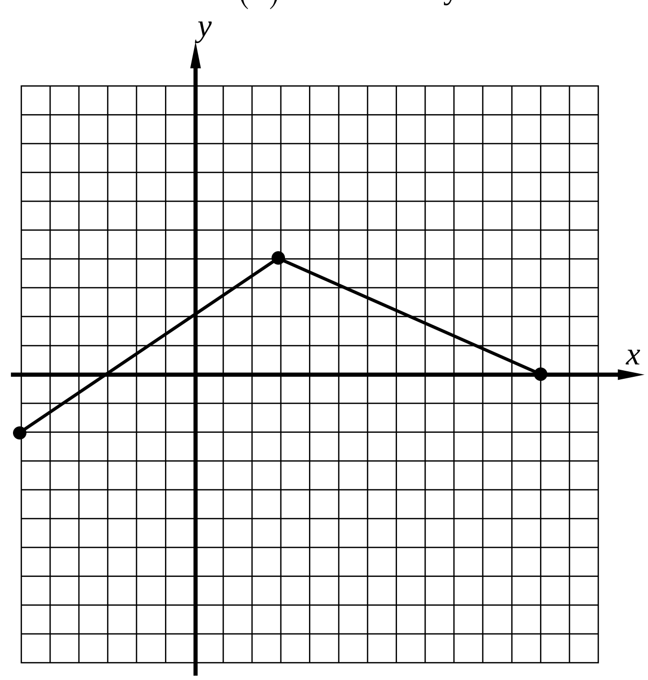
x									
y									

(b) What was the effect on the graph of $f(x)$ when we multiplied the input by $\frac{1}{2}$?

We can certainly combine the effects of both a vertical stretch and a horizontal stretch. This is harder, but if you can identify the various transformations, then the new function's graph can often be produced from the older function's fairly easily.

***Exercise* #3:** The graph of $f(x)$ is shown on the grid below. A new function $h(x)$ is defined by:

$$h(x)=2f(3x)$$



(a) Evaluate $h(1)$. What point must lie on the graph of $h(x)$ based on this calculation?

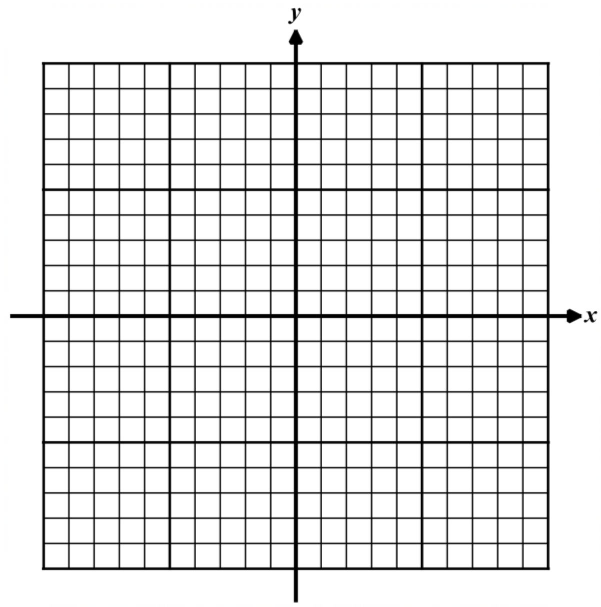
(b) Describe the transformations that must be done to the graph of $f(x)$ to produce the graph of $g(x)$.

(c) Graph $g(x)$ by plotting the three major points.

2-5 MORE WORK WITH PIECEWISE FUNCTIONS

So far, we have worked with **linear piecewise defined functions**, or those functions whose outputs are defined using different linear formulas over different parts of its domain. We certainly don't have to be confined to just linear equations, though. In this lesson, we will examine more complicated functions.

***Exercise* #1:** Consider the function $f(x)=\begin{cases} 2x+5 & x\le 0 \\ \frac{1}{2}x^2-4 & x>0 \end{cases}$.

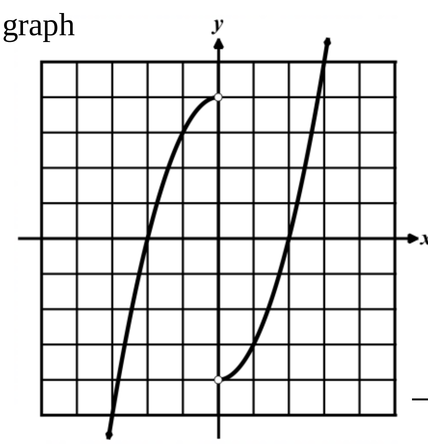


(a) Plot this function on the grid provided. Show a table of values below.

(b) Why is the average rate of change of $f(x)$ constant to the left of the origin but not constant to the right of the origin?

When looking at any piecewise function, be sure to recall what you know about basic **linear** and **quadratic** functions.

***Exercise* #2:** Which of the following formulas properly models the graph shown to the right?



(1) $f(x)=\begin{cases} x^2-4 & x<0 \\ x^2+4 & x>0 \end{cases}$

(3) $f(x)=\begin{cases} x^2-4 & x<0 \\ 4-x^2 & x>0 \end{cases}$

(2) $f(x)=\begin{cases} 4-x^2 & x<0 \\ x^2-4 & x>0 \end{cases}$

(4) $f(x)=\begin{cases} x^2+4 & x<0 \\ x^2-4 & x>0 \end{cases}$

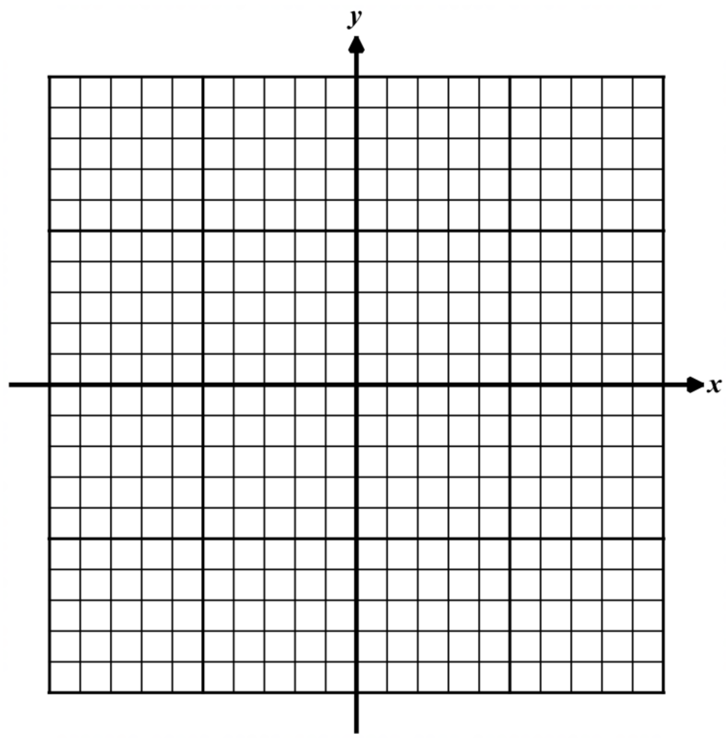
____

Piecewise functions can be intimidating to work. Just keep in mind the various domains over which the formulas apply.

***Exercise* #3:** Two functions $f(x)$ and $g(x)$ are defined below.

$$f(x)=\begin{cases}\sqrt{x+4}-1 & -4\le x\le 5\\ -\dfrac{1}{2}x+\dfrac{9}{2} & x>5\end{cases} \quad \text{and} \quad g(x)=x-6$$

(a) Graph and label the two functions on the grid.

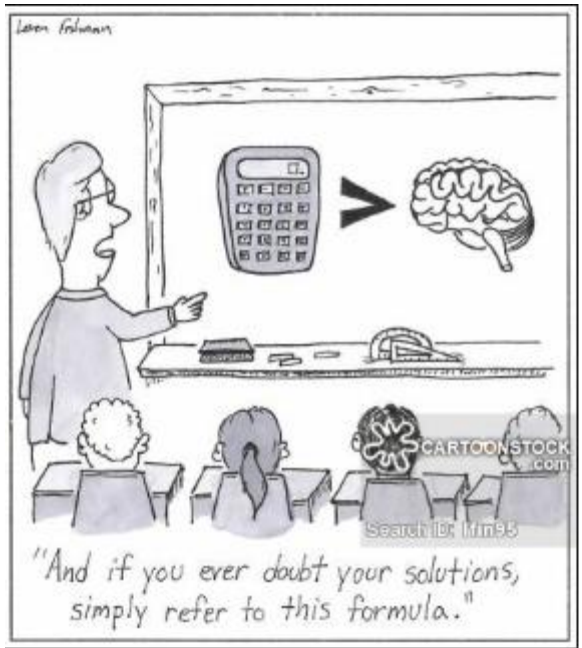


(b) What is the maximum value of $f(x)$?

(c) State all zeros of $f(x)$.

(d) What values of x solve the equation $f(x)=g(x)$. Use your graph to justify your response.

(e) Over what interval is the average rate of change of $f(x)$ the same as that of $g(x)$? Explain how you arrived at your interval.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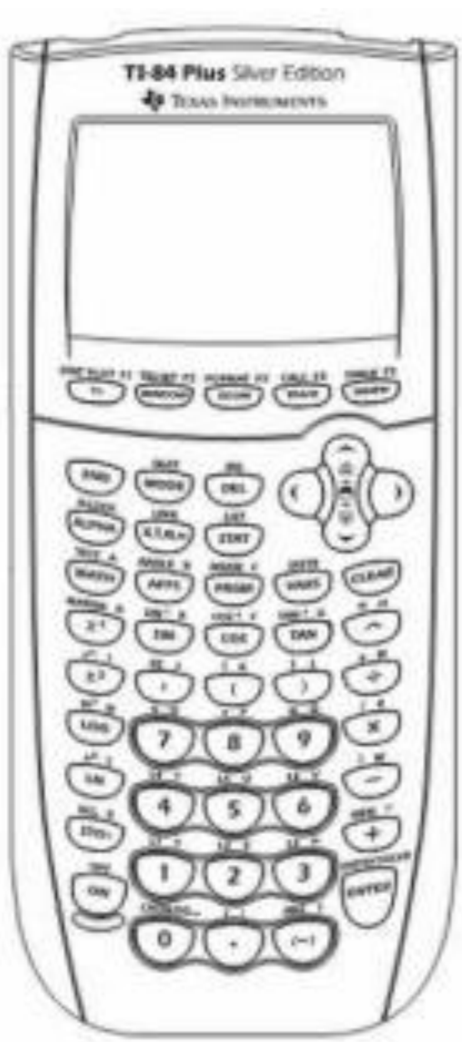
"And if you ever doubt your solutions, simply refer to this formula."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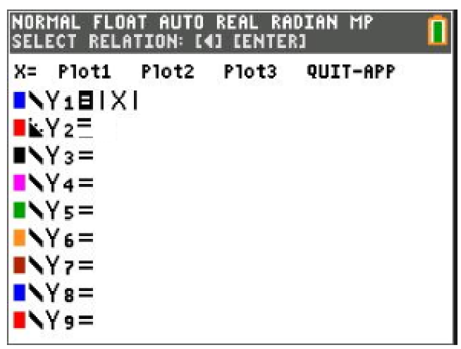
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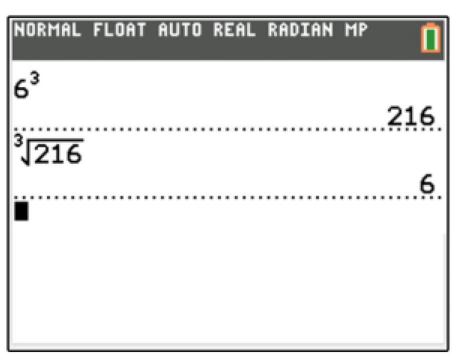
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