Name: ___________________________________ Date: _________________

2-6 Complete Factoring, Grouping, Cubes

Each expression that we have factored has been the product of two quantities. But, factoring can produce many more than just two factors. In *Exercise* #1, we first warm-up by multiplying three factors together.

***Exercise* #1:** Write each of these in their simplest form. The last two should take little time to do.

(a) $2(x+4)(x+7)$ (b) $5(2x-5)(x+3)$ (c) $3(x-5)(x+5)$ (d) $4x(3x-2)(3x+2)$

To completely factor an expression means to write it as a product which includes binomials that contain no greatest common factors (gcf's).

***Exercise* #2:** Consider the trinomial $2x^2-4x-6$.

(a) Verify that both of the following products are *correct* factorizations of this trinomial.

$(2x-6)(x+1)$ $(2x+2)(x-3)$

(b) Why are neither of these completely factored?

(c) Write each of these in completely factored form by factoring out the gcf of each unfactored binomial.

(d) What is true of both complete factorizations you found in part (c)?

In practicality, it is always easiest to completely factor by looking for a gcf of the expression first. Once removed, the factoring then either consists of the difference of perfect squares or standard trinomial techniques.

***Exercise* #3:** Write each of the following in its completely factored form. These should be relatively easy.

(a) $4x^2+12x-40$ (b) $6x^2-24$ (c) $2x^2+20x+50$ (d) $75-3x^2$

2-6 FACTORING BY GROUPING & CUBES

You now have essentially three types of factoring: (1) greatest common factor, (2) difference of perfect squares, and (3) trinomials. We can combine gcf factoring with the other two to **completely factor** quadratic expressions. Today we will introduce a new type of factoring known as **factoring by grouping**. This technique requires you to **see structure in expressions**.

***Exercise* #1:** Factor a binomial common factor out of each of the following expressions. Write your final expression as the product of two binomials.

(a) $x(2x+1)+7(2x+1)$

(b) $5x(x-2)-4(x-2)$

(c) $(x+5)(x-7)+(x-7)(x+1)$

(d) $(2x+8)(x+4)-(x-2)(x+4)$

Some **very special** polynomials can be factored by taking advantage of the structure we have seen in the last two problems. The key is to do **mindful manipulations** of expressions so that they **remain equivalent** but are written as an overall product. When we **factor by grouping** we first extract common factors from pairs of binomials in four-term polynomials. If we are **lucky** we are left with another **binomial common factor**.

***Exercise* #4:** Use the method of factoring by grouping to completely factor the following expressions.

(a) $3x^3+2x^2-27x-18$

(b) $18x^3+9x^2-2x-1$

(c) $x^5+4x^3+2x^2+8$

(d) $5x^3+10x^2+20x+40$

2-6 FACTORING SUM AND DIFFERENCE OF PERFECT CUBES

In algebra there are a variety of polynomial identities – these are equations that are true regardless of the value of the variables. One important identity you have already discovered is the product of conjugate binomials. Most of you may recognize this as the difference of perfect squares.

MULTIPLYING CONJUGATE PAIRS

$$(a+b)(a-b)=a^2-b^2$$

Exercise 1: Verify each of the following difference of perfect squares.

(a) $(x+5)(x-5)$

(b) $(2x+3)(2x-3)$

Just as there are patterns for the difference of perfect squares, there is also a pattern for the sum and difference for perfect cubes.

SUM OF PERFECT CUBES

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

DIFFERENCE OF PERFECT CUBES

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

Exercise 2: Multiple each of the following to verify the polynomial identity.

(a) $(a+b)(a^2-ab+b^2)$

(b) $(a-b)(a^2+ab+b^2)$

Exercise 3: Factor each of the following expressions using perfect cubes.

Sum or Difference of Cubes
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
OR
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

If factoring two terms that are perfect cubes, we can apply the sum or difference of cubes rule to help us factor.

1) Make two sets of parenthesis and put the cube root of each term in the first one and keep the sign
2) Now work just with the first parenthesis to fill in the second set of parenthesis:
   a) Square first term
   b) Multiply two terms and change the sign
   c) Square last term, using a positive sign

(a) $x^3 - 8$
Let a =
Let b =

(b) $27x^3 + 1$
Let a =
Let b =

(c) $x^3y^3 - 64$

(d) $125x^6 + 8$

(e) $5x^3 + 40$

(f) $16x^3 - 54$

2-7 REVIEW ZERO PRODUCT LAW

One of the most important equation solving technique stems from a fact about the number zero that is **not true of any other number**:

> **THE ZERO PRODUCT LAW**
>
> If the **product** of multiple factors is **equal to zero** then at least **one of the factors must be equal to zero**.

The law can immediately be put to use in the first exercise. In this exercise, quadratic equations are given already in factored form.

***Exercise* #1:** Solve each of the following equations for all value(s) of *x*.

(a) $(x+7)(x-3)=0$ (b) $(2x-5)(x-4)=0$ (c) $4(3x+2)(4x-3)=0$

***Exercise* #2:** In *Exercise* #1(c), why does the factor of 4 have no effect on the solution set of the equation?

The Zero Product Law can be used to solve any quadratic equation that is factorable (not prime). To utilize this technique the problem solver must first set the equation equal to zero and then factor the non-zero side.

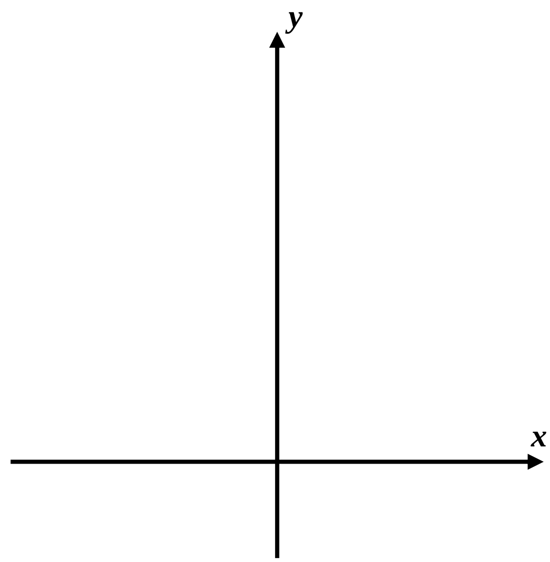
***Exercise* #3:** Solve each of the following quadratic equations using the Zero Product Law.

(a) $x^2+3x-14=-2x+10$ (b) $3x^2+12x-7=x^2+3x-2$

***Exercise* #4:** Consider the system of equations shown below consisting of a parabola and a line.

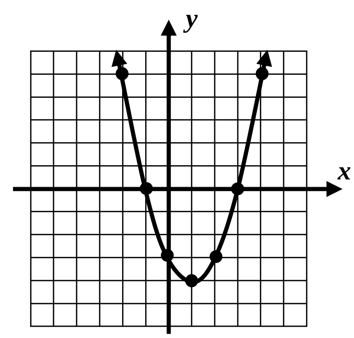
$y = 3x^2 - 8x + 5$ and $y = 4x + 5$

(a) Find the intersection points of these curves *algebraically*.



The Zero Product Law is extremely important in finding the **zero's** or ***x*-intercepts (zeroes)** of a parabola.

***Exercise* #5:** The parabola shown at the right has the equation $y = x^2 - 2x - 3$.



(a) Write the coordinates of the two *x*-intercepts of the graph.

(b) Find the *x*-intercepts of this parabola *algebraically*.

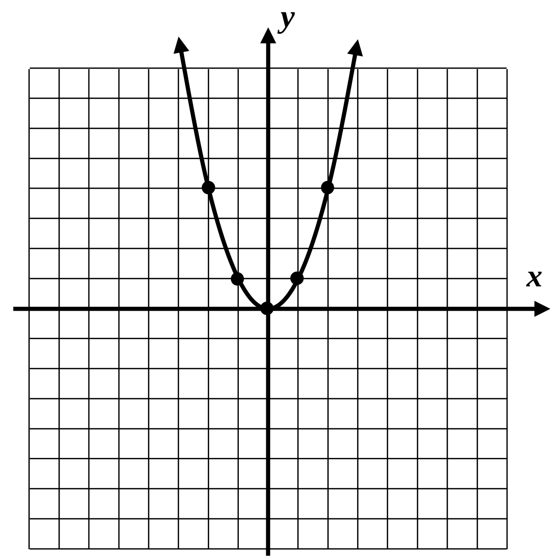
***Exercise* #6:** *Algebraically* find the set of *x*-intercepts (zeroes) for each parabola given below.

(a) $y = 4x^2 - 1$ (b) $y = 3x^2 + 13x - 10$ (c) $y = 5x^2 - 10x$

2-8 MORE COMPLETING THE SQUARE AND SHIFTING PARABOLAS

Parabolas, and graphs more generally, can be moved horizontally and vertically by simple manipulations of their equations. This is known as **shifting** or **translating** a graph. You worked with this extensively in Common Core Algebra I. The first exercise will review how to use a method known as **completing the square** to identify shifts and the turning point of a parabola.

***Exercise* #1:** The function $y=x^2$ is shown already graphed on the grid below. Consider the quadratic whose equation is $y=x^2-8x+18$.



(a) Using the method of completing the square, write this equation in the form $y=(x-h)^2+k$.

(b) Describe how the graph of $y=x^2$ would be shifted to produce the graph of $y=x^2-8x+18$.

(c) Sketch the graph of $y=x^2-8x+18$ by using its **vertex form** in (a). What are the coordinates of its turning point (vertex)?

The **algorithm** of completing the square works best when $a=1$ and b is even in the form $y=ax^2+bx+c$.

x^2-6x+2

x^2+2x+5

But, it does work in every case, even the messy ones.

***Exercise* #3:** Place each of the following quadratic functions in vertex form and identify the turning point.

(a) $y = 3x^2 + 12x - 2$

(b) $y = 2x^2 + 6x + 1$

***Exercise* #4:** The method of completing the square can be performed on the standard quadratic equation $y = ax^2 + bx + c$ and after much manipulation can be placed in the form:

$$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

(a) Based on this formula, what is the x-coordinate of the turning point of any parabola? Be careful.

(b) Use this formula to find the turning point of the parabola $y = x^2 + 10x - 2$.

(c) Verify your answer from part (a) by placing the quadratic $y = x^2 + 10x - 2$ into vertex form.

(d) Verify both answers by examining a table on your calculator using the original equation.

***Exercise* #5:** Use the formula $x = -\frac{b}{2a}$ to find the turning points for each of the following quadratic functions.

(a) $f(x) = 2x^2 - 12x + 7$

(b) $g(x) = -\frac{1}{4}x^2 + 5x - 20$