Name: **Answer Key** Date: _____

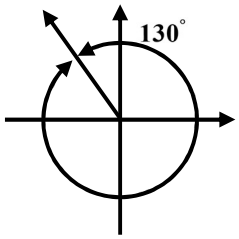
UNIT 7 – TRIGONOMETRY REVIEW
MATH III

Part I Questions

1. Which of the following angles is coterminal with an angle of 130°, assuming both angles are drawn in the standard position?

   (1) 230° (3) 430°

   (2) -230° (4) -310°



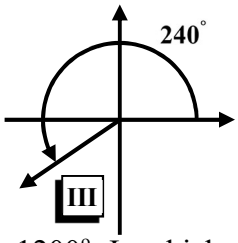
$360^\circ - 130^\circ = 230^\circ$ clockwise $\Rightarrow -230^\circ$

(2)

2. If drawn in the standard position, which of the following angles terminates in the third quadrant?

   (1) 120° (3) -210°

   (2) -60° (4) 240°



(4)

3. A rotation angle, drawn in standard position, measures 1200°. In which quadrant does its terminal ray lie?

   (1) I (3) III

   (2) II (4) IV

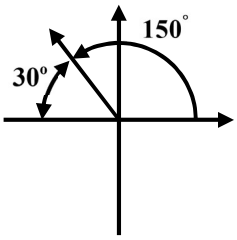
$1200^\circ - 3(360^\circ) = 120^\circ \Rightarrow$ II

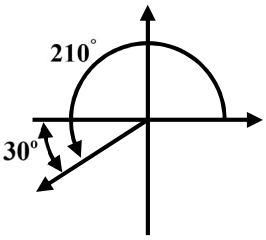
(2)

4. Which of the following has the same reference angle as 150°?

   (1) 210° (3) 120°

   (2) 300° (4) 70°





(1)

5. The radian angle $\frac{3\pi}{4}$ is equivalent to

   (1) 67.5° (3) 270°

   (2) 135° (4) 325°

$\frac{3\pi}{4} \times \frac{180^\circ}{\pi} = \frac{3}{4}(180) = 135^\circ$

(2)

6. The angle 240° can be written equivalently as which of the following in the radian system?

(1) $\frac{7\pi}{6}$ (3) $\frac{3\pi}{2}$

(2) $\frac{5\pi}{4}$ (4) $\frac{4\pi}{3}$

$$240^\circ \times \frac{\pi}{180^\circ} = \frac{240\pi}{180} = \frac{24\pi}{18} = \frac{4\pi}{3}$$

(4)

7. If the minute hand of a clock is 3.5 inches long and its tip rotates through a distance of 8 inches, then which of the following is closest to the angle that it rotates?

(1) 131° (3) 267°

(2) 174° (4) 314°

$$\theta_{\text{rad}} = \frac{s}{r} = \frac{8}{3.5} = 2.2857 \text{ radians}$$
$$2.2857 \times \frac{180^\circ}{\pi} = 130.96... \approx 131^\circ$$

(1)

8. A goat is attached to a 12 foot long leash pulled tight and rotates through an angle of 230°. Which of the following is closest to the distance that the goat travels?

(1) 23 ft (3) 41 ft

(2) 32 ft (4) 48 ft

$$\frac{s}{24\pi} = \frac{230^\circ}{360^\circ} \Rightarrow s = \frac{230}{360} \cdot 24\pi = 48.171... \approx 48 \text{ ft}$$

(4)

9. A point lies on the unit circle whose x-coordinate is $\frac{1}{4}$. If the point lies in the fourth quadrant, then which of the following is its y-coordinate?

(1) $\frac{3}{4}$ (3) $-\frac{\sqrt{7}}{4}$

(2) $-\frac{\sqrt{15}}{4}$ (4) $\frac{\sqrt{11}}{2}$

$$x^2 + y^2 = 1$$
$$\left(\frac{1}{4}\right)^2 + y^2 = 1$$
$$y^2 + \frac{1}{16} = 1$$
$$y^2 = \frac{15}{16}$$
$$y = \pm\sqrt{\frac{15}{16}} = \pm\frac{\sqrt{15}}{\sqrt{16}} = \pm\frac{\sqrt{15}}{4}$$

We are told that the point lies in the fourth quadrant. In the fourth quadrant, the y-coordinates are negative. Thus, the correct answer is:

$$y = -\frac{\sqrt{15}}{4}$$

(2)

10. The terminal ray of an angle drawn in standard position on the unit circle that measures 30° has coordinates of $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. Based on this information, what is the value of $\cos\left(150^\circ\right)$?

(1) $\frac{1}{2}$ (3) $\frac{\sqrt{3}}{2}$

(2) $-\frac{1}{2}$ (4) $-\frac{\sqrt{3}}{2}$

The angle 150° has a reference angle of 30°, and so its cosine must have the same absolute value. Since cosine is the *x*-coordinate on the unit circle and in the second quadrant, where 150° is located, the *x*-coordinate is negative:

$$\cos\left(150^\circ\right) = -\frac{\sqrt{3}}{2}$$

(4)

11. For an angle A that terminates in the second quadrant, $\sin A = \frac{2}{3}$. Which of the following calculations would result in the value of $\cos A$?

(1) $\sqrt{1+\left(\frac{2}{3}\right)^2}$ (3) $-\sqrt{1-\left(\frac{2}{3}\right)^2}$

(2) $-\sqrt{1+\left(\frac{2}{3}\right)^2}$ (4) $\sqrt{1-\left(\frac{2}{3}\right)^2}$

$$\cos^2 A + \sin^2 A = 1$$
$$\cos^2 A + \left(\frac{2}{3}\right)^2 = 1$$
$$\cos^2 A = 1 - \left(\frac{2}{3}\right)^2$$
$$\cos A = \pm\sqrt{1-\left(\frac{2}{3}\right)^2}$$

In the second quadrant, cosine is negative and thus:

$$\cos A = -\sqrt{1-\left(\frac{2}{3}\right)^2}$$

(3)

12. Which of the following could *not* be the value of the cosine of an angle?

(1) $-\frac{4}{5}$ (3) $\frac{\sqrt{11}}{4}$

(2) $\frac{7}{3}$ (4) $-\frac{\sqrt{3}}{2}$

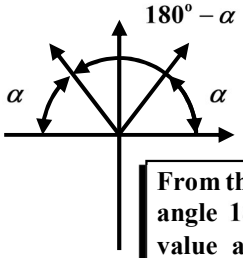
All values of cosine (and sine) must be between -1 and 1 (inclusive). Hence, because $\frac{7}{3}$ is larger than 1, it cannot be a cosine value.

(2)

13. If α is an angle such that $0^\circ < \alpha < 90^\circ$ and $\sin\alpha = 0.56$ then which of the following is the value of $\sin\left(180^\circ - \alpha\right)$?

(1) 0.56 (3) 0.83

(2) –0.56 (4) –0.83



From this diagram, we can tell that the angle $180 - \alpha$ will have the same sine value as the angle α because they would be at the same *y*-value on the unit circle.

(1)

14. If $f(x)=10\sin(2x)+8$ then $f\left(\frac{\pi}{4}\right)=$?

(1) $4\sqrt{2}$ (3) 18

(2) 8 (4) $28\sqrt{3}$

$$f\left(\frac{\pi}{4}\right)=10\sin\left(2\cdot\frac{\pi}{4}\right)+8$$
$$=10\sin\left(\frac{\pi}{2}\right)+8$$
$$=10(1)+8$$
$$=18$$

(3)

15. If an angle has a positive cosine but a negative sine then it must terminate in which of the following quadrants?

(1) I (3) III

(2) II (4) IV

$\cos A > 0 \Rightarrow$ I or IV
and
$\sin A < 0 \Rightarrow$ III or IV

Therefore, the angle must be located in Quadrant IV to make both of these conditions true.

(4)

16. Which of the following represents the range of the function $y=-6\sin(x)+10$?

(1) $-60 \le y \le 60$ (3) $-16 \le y \le 4$

(2) $0 \le y \le 20$ (4) $4 \le y \le 16$

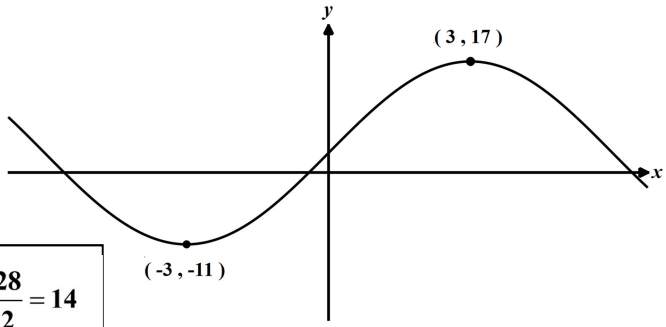
$y_{min} = 10-6=4$
$y_{max} = 10+6=16$
Range: $4 \le y \le 16$

(4)

17. Given the sinusoidal graph with coordinates shown below, which of the following is the value of its amplitude?

(1) 14 (3) 12

(2) 6 (4) 28



$$\text{Amplitude} = \frac{y_{max}-y_{min}}{2} = \frac{17-(-11)}{2} = \frac{28}{2} = 14$$

(1)

18. A periodic function has an equation $y=10\cos(8x)-2$. What is the horizontal distance between any two consecutive relative maximums on this graph?

(1) 10 (3) 8

(2) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

Asking for the period:
$BP = 2\pi$
$8P = 2\pi$
$P = \frac{2\pi}{8} = \frac{\pi}{4}$

(4)

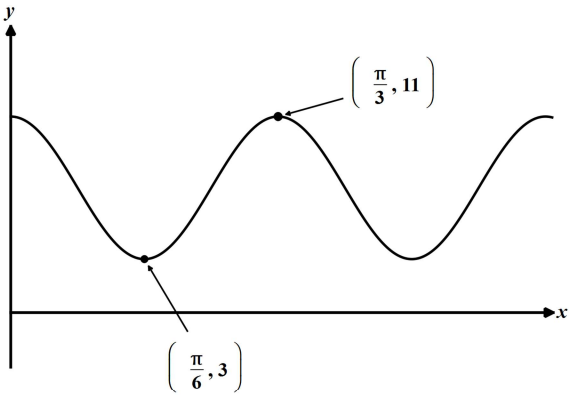
19. The graph shown below can be described using the equation $y = A\cos(Bx) + k$. Which of the following is the value of $B + k$?

(1) 5π

(2) 13

(3) 11

(4) $\frac{\pi}{7}$



$P = \frac{\pi}{3} \Rightarrow \frac{\pi}{3}B = 2\pi \Rightarrow B = \frac{3}{\pi} \cdot 2\pi = 6$

$k = \frac{3+11}{2} = \frac{14}{2} = 7$

$B + k = 13$

(2)

20. Which of the following lines would the graph of $y = -5\sin(x) + 14$ *not* intersect?

(1) $x = 0$

(2) $x = \pi$

(3) $y = 20$

(4) $y = 9$

Determine the Range:

$y_{min} = 14 - 5 = 9$

$y_{max} = 14 + 5 = 19$

$9 \le y \le 19$

(3)

21. A person riding a Ferris wheel at a local fair makes one complete trip around in 10 minutes. Their height can be modeled using a sine function of the form $y = A\sin(Bt) + C$, where t is the amount of time the person has been traveling, in minutes. Which of the following must be the value of B?

(1) 10

(2) $\frac{1}{20}$

(3) 10π

(4) $\frac{\pi}{5}$

$BP = 2\pi$

$P = 10$ minutes

$10B = 2\pi$

$B = \frac{2\pi}{10} = \frac{\pi}{5}$

(4)

22. The volume of water in a tank varies periodically. At $t = 0$ it is at its maximum of 650 gallons and at $t = 5$ it is at its minimum of 120 gallons. Which of the following functions would best model the volume of water in this tank as a function of time in hours?

(1) $V = 265\cos\left(\frac{2\pi}{10}t\right) + 385$

(2) $V = -770\sin(10t) + 385$

(3) $V = -385\cos(5t) + 265$

(4) $V = 265\sin\left(\frac{\pi}{10}t\right) + 770$

$\text{Amplitude} = \frac{650 - 120}{2} = \frac{530}{2} = 265$

$\text{midline} = \frac{120 + 650}{2} = \frac{770}{2} = 385$

$\text{period} = 10 \text{ min} \Rightarrow B = \frac{2\pi}{10}$

(1)

23. The terminal ray of an angle drawn in standard position passes through the point $(.508,.862)$ on the unit circle. Which of the following is closest to the tangent of this angle?

(1) .685 (3) 1.697

(2) 1.291 (4) 2.883

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{y-coordinate}}{\text{x-coordinate}} = \frac{.862}{.508} = 1.69685... \approx 1.697$$

(3)

24. If α is an angle drawn in the standard position with its terminal ray landing in the fourth quadrant and $\csc(\alpha) = -5$, then which of the following is the exact value of $\cos(\alpha)$?

(1) $-\frac{1}{5}$ (3) $\frac{\sqrt{24}}{5}$

(2) $-\frac{24}{25}$ (4) $\frac{\sqrt{6}}{2}$

$$\csc\alpha = -5 \Rightarrow \sin\alpha = -\frac{1}{5}$$
$$\cos^2\alpha + \sin^2\alpha = 1$$
$$\cos^2\alpha + \left(-\frac{1}{5}\right)^2 = 1$$
$$\cos^2\alpha + \frac{1}{25} = 1$$
$$\cos^2\alpha = \frac{24}{25}$$

$$\cos\alpha = \pm\sqrt{\frac{24}{25}} = \pm\frac{\sqrt{24}}{5}$$

in the fourth quadrant cosine is positive so...

$$\cos\alpha = \frac{\sqrt{24}}{5}$$

(3)

25. For the angle θ it's known that $\cot(\theta) < 0$ and $\sin(\theta) > 0$. In which quadrant does the terminal ray of θ lie?

(1) I (3) III

(2) II (4) IV

$\cot\theta < 0 \Rightarrow$ II or IV
and
$\sin\theta > 0 \Rightarrow$ I or II
thus must be II

(2)

Free Response Questions

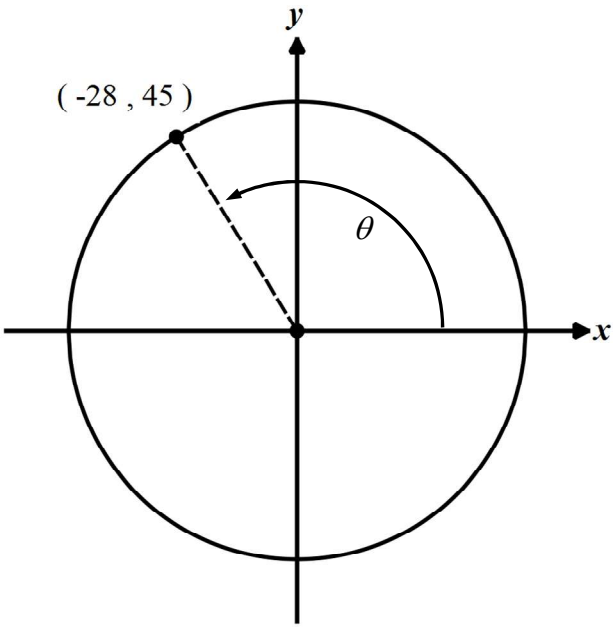
26. An angle drawn in standard position measures 10 radians. In what quadrant does its terminal ray lie? Show the reasoning that leads to your answer.

$$10 \text{ radians} \times \frac{180^\circ}{\pi} = 572.96^\circ$$

Since this angle is greater than 360°, we need to subtract 360° from the angle to pass one full rotation.

$572.96^\circ - 360^\circ = 212.96^\circ$
Quadrant III

27. Given the following circle (note that it is **not** the **unit circle**) with the angle θ marked, state the values of each of the following:

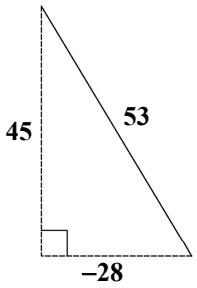


(a) The radius of the circle

$28^2 + 45^2 = r^2 \Rightarrow r^2 = 2809 \Rightarrow r = \sqrt{2809} = 53$

(b) $\sin\theta =$ $\frac{45}{53}$

(c) $\cos\theta =$ $-\frac{28}{53}$



(d) $\tan\theta =$ $\frac{45}{-28} = -\frac{45}{28}$

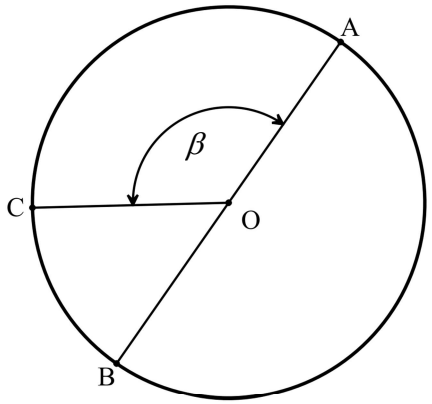
(e) $\sec\theta =$ $\frac{1}{\cos\theta} = -\frac{53}{28}$

(f) $\csc\theta =$ $\frac{1}{\sin\theta} = \frac{53}{45}$

(g) $\cot\theta =$ $\frac{1}{\tan\theta} = -\frac{28}{45}$

28. In the circle shown below, $AB = 30$ and the length of the minor arc from point A to point C is 40. Find the exact measure of the marked angle β in terms of radians. Show how you arrived at your answer.

Since the diameter = 30
$r = 15$



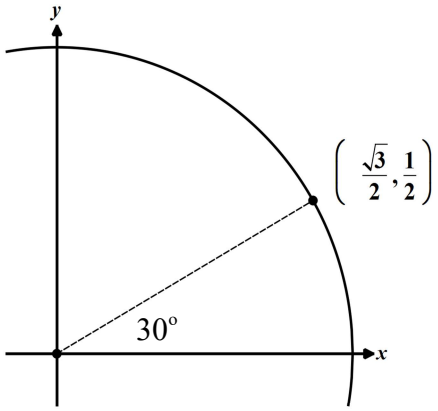
$\theta_{rad} = \frac{s}{r} = \frac{40}{15} = \frac{8}{3}$

29. For an angle A it is known that $\sin A = \frac{3}{4}$ and $\cos A < 0$. Determine the value of $\tan A$. Show how you arrived at your answer.

$$\cos^2 A + \sin^2 A = 1$$
$$\cos^2 A + \left(\frac{3}{4}\right)^2 = 1$$
$$\cos^2 A + \frac{9}{16} = 1$$
$$\cos^2 A = \frac{7}{16}$$
$$\cos A = -\sqrt{\frac{7}{16}} = -\frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\sin A}{\cos A}$$
$$= \frac{\frac{3}{4}}{-\frac{\sqrt{7}}{4}} = \frac{3}{\cancel{4}} \cdot -\frac{\cancel{4}}{\sqrt{7}} = -\frac{3}{\sqrt{7}} \quad \text{or} \quad -\frac{3\sqrt{7}}{7}$$

30. A portion of the unit circle is shown below. Based on this information, determine the value of $\sec(150^\circ)$ in exact form. Explain how you arrived at your answer.



Note that 150° has a reference angle of 30°.

$$\sec(150^\circ) = \frac{1}{\cos(150^\circ)}$$
$$= \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$$

31. For the function $f(x) = 6\sin(10x) + 8$, explain why the equation $f(x) = 0$ would fail to have any solutions.

$$f_{\min} = 8 - 6 = 2$$
$$f_{\max} = 8 + 6 = 14$$
Range of $f(x)$: $[2, 14]$

Since the range of this function does not contain 0, the function will never have an output of 0. Therefore, the equation $f(x) = 0$ will have no solutions.

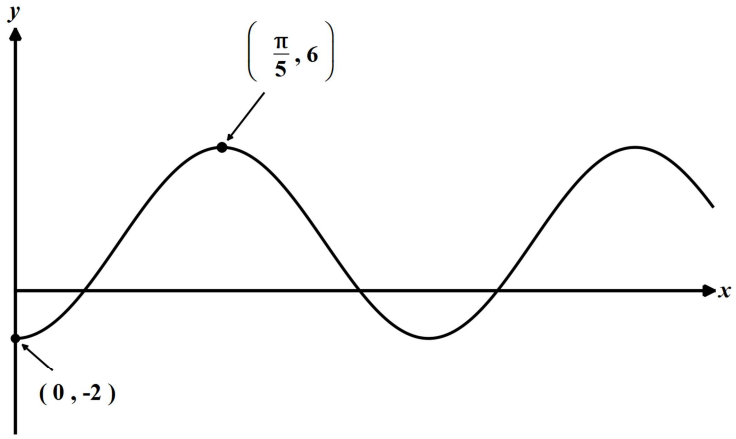
32. For the function $f(x) = A\sin\left(\frac{\pi}{5}x\right) + k$, it is known that $f(3) = 7$. Explain why $f(13)$ must also equal 7.

$$BP = 2\pi$$
$$\frac{\pi}{5}P = 2\pi$$
$$P = 2\cancel{\pi} \cdot \frac{5}{\cancel{\pi}} = 10$$

Because the period of this function is 10, it means that every 10 horizontal units, the pattern will repeat itself. In fact, it means:

$$f(x+10) = f(x)$$
$$f(13) = f(3+10) = f(3) = 7$$

33. The graph shown below can be modeled using the equation $y = A\cos(Bx) + C$. Determine the values of A, B, and C. Show how you arrived at your results.



$|A| = \text{amplitude} = \dfrac{6-(-2)}{2} = 4$

$A = -4$ because this cosine graph starts at its minimum

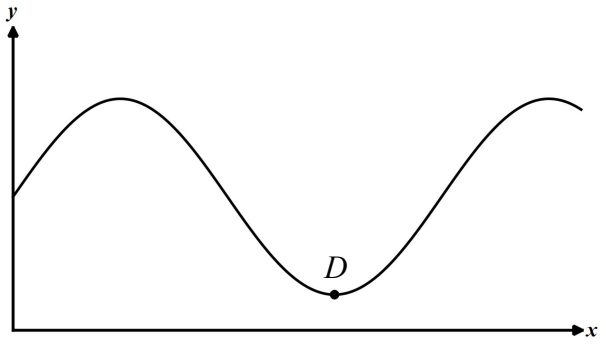
$C = \dfrac{6+-2}{2} = \dfrac{4}{2} = 2$

$P = 2\left(\dfrac{\pi}{5}\right) = \dfrac{2\pi}{5}$

$BP = 2\pi \Rightarrow \dfrac{2\pi}{5}B = 2\pi$

$B = 5$

34. If the function $y = A\sin\left(\dfrac{\pi}{8}x\right) + C$ is graphed below, answer the following questions about point D marked.



(a) What is the numerical value of the x-coordinate of point D? Show how you arrived at your answer.

$\dfrac{\pi}{8}P = 2\pi$

$P = 2\not{\pi} \cdot \dfrac{8}{\not{\pi}} = 16$

But point D is only $\frac{3}{4}$ of a period away from the y-axis. So, it's x-coordinate is $\frac{3}{4}(16) = 12$.

(b) What is the y-coordinate of D in terms of the constants A and C?

$y_{\min} = C - A$

35. A person's height, in feet above the ground, on a Ferris wheel can be modeled using the equation $h(t) = -45\cos\left(\dfrac{\pi t}{7}\right) + 52$, where t is the time the rider has been on the wheel in minutes. What is the maximum height the rider reaches and the time it takes to first reach this height if they get on at $t = 0$. Explain how you arrived at your answer.

$h_{\max} = 52 + 45 = 97$ feet

$\dfrac{\pi}{7}P = 2\pi$

$P = 2\not{\pi} \cdot \dfrac{7}{\not{\pi}} = 14$ minutes

⇨

Since it only takes one-half of a period to reach the top height from the bottom of the Ferris wheel, the time it first reaches this height will be at 7 minutes.