# 1-1 REVIEW OF FUNCTIONS

A ___________ is any "rule" that assigns exactly one __________ (*domain*) for each __________ value (*range*). These rules can be expressed in different ways, the most common being equations, graphs, and tables of values. We call the input variable **independent** and output variable **dependent**.

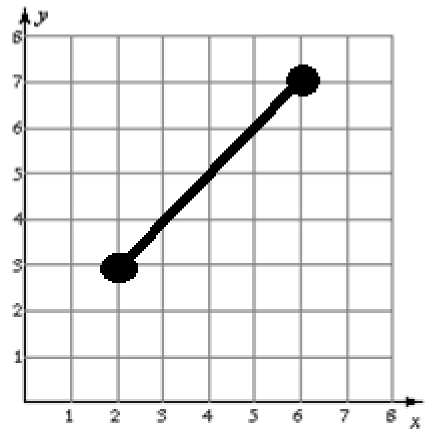
**Find the domain and range:**

| x | 3 | 4 | 5 | 6 |
|---|---|---|---|---|
| y | 1 | 2 | 2 | 3 |

D: { }

R: { }

**Find the domain and range:**



D:

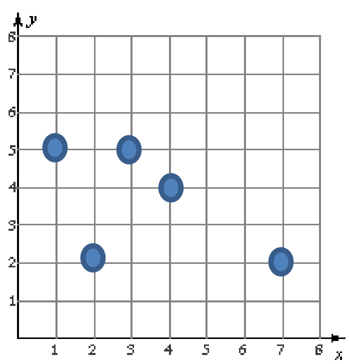
R:

**Examples of _______________:**

**1.**

| x | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| y | 3 | 4 | 4 | 6 |

**2.**

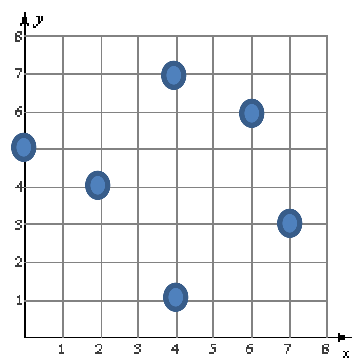


**Examples of ___________________:**

**3.**

| x | 5 | 6 | 6 | 7 |
|---|---|---|---|---|
| y | 1 | 2 | 3 | 4 |

**4.**



How can you tell if something is a function?!

Indicate whether each is a function or not a function. Then give the domain and range.

1. f(x) = {(3, 5), (7, 4), (6, 4), (-3, 1), (-2, -2)}

   Function? __________ Domain = __________________________ Range= ___________________________

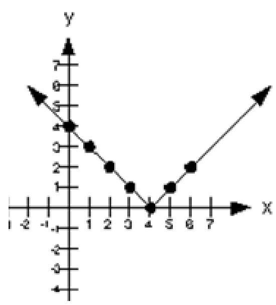
2. g(x) = {(1, 3), (0, 9), (10, 12), (1, 15)}

   Function? __________ Domain = __________________________ Range= ___________________________

3. h(x) = 3x – 7

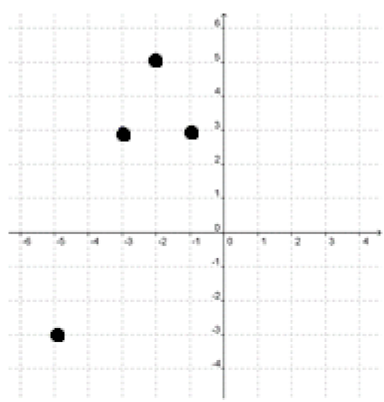
   Function? __________ Domain = __________________________ Range= ___________________________

4. m(x) =




   Function? __________ Domain = __________________________ Range= ___________________________

5. r(x) =



   Function? __________ Domain = __________________________ Range= ___________________________

Use the definitions in problems 1-5 to find these values:

6. f(6) = _________ 7. g(1) = __________ 8. h(5) = ___________ 9. h(a) = __________

10. m(5) = ________ 11. r(-1) = _________ 12. 2g(0) – 4m(1) = ___________________________

13. If f(x) = 4, then x = __________ 14. If m(x) = 2, then x = _______________

# Function Notation & Evaluating a Function

**Function Notation**

$$y = f(x)$$

Output → $y$  Rule → $f$  Input → $x$

1. Without using your calculator, evaluate each of the following given the function definitions and input values.

(a) $f(x) = 3x + 7$

$f(-4) =$

$f(2) =$

(b) $g(x) = 3x^2$

$g(2) =$

$g(-3) =$

(c) $h(x) = \sqrt{x-5}$

$h(41) =$

$h(14) =$

2. Evaluate each of the following more complex functions.

(a) $f(x) = \dfrac{3x^2 - 5}{4x + 10}$

$f(-5) =$

$f(0) =$

(b) $g(x) = \dfrac{\sqrt{25 - x^2}}{x}$

$g(4) =$

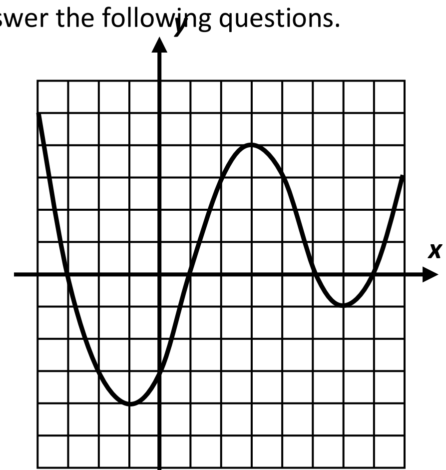
$g(-3) =$

(c) $h(x) = 30(1.2)^x$

$h(3) =$

$h(0) =$

3. Based on the graph of the function $y = g(x)$ shown below, answer the following questions.



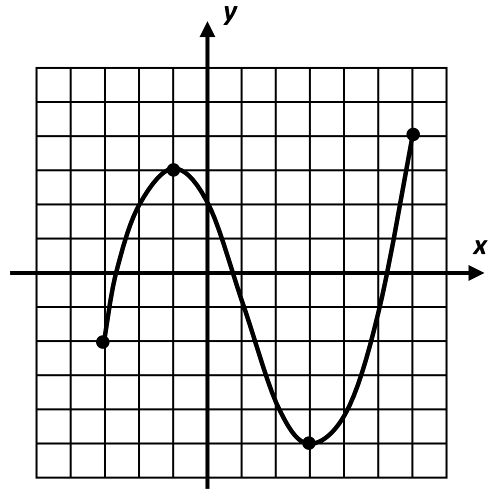

(a) Evaluate $g(-2)$, $g(0)$, $g(3)$ and $g(7)$.

(b) What values of *x* solve the equation $g(x) = 0$

(c) Graph the horizontal line $y = 2$ on the grid above and label.

(d) How many values of *x* solve the equation $g(x) = 2$?

4. The function $y = g(x)$ is completely defined by the graph shown below. Answer the following questions based on this graph.



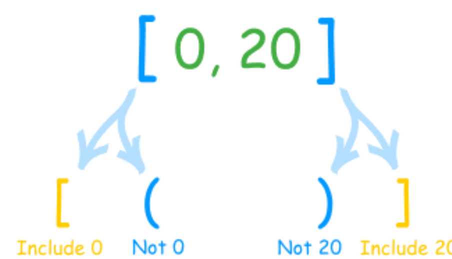

(a) Determine the minimum and maximum $x$-values represented on this graph.

(b) Determine the minimum and maximum $y$-values represented on this graph.

(c) State the domain and range of this function using set builder notation.

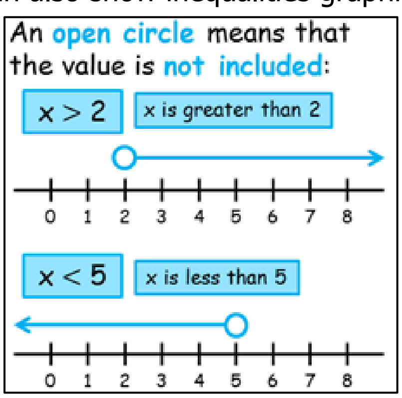
# Symbol Review

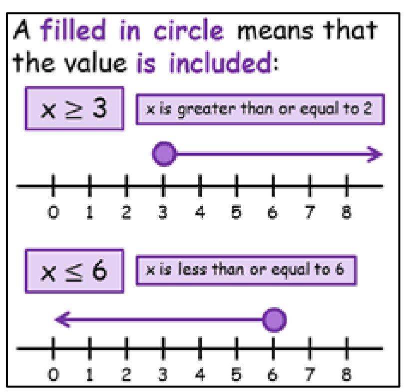
In _______________ we just write the beginning and ending numbers of the interval, and use:

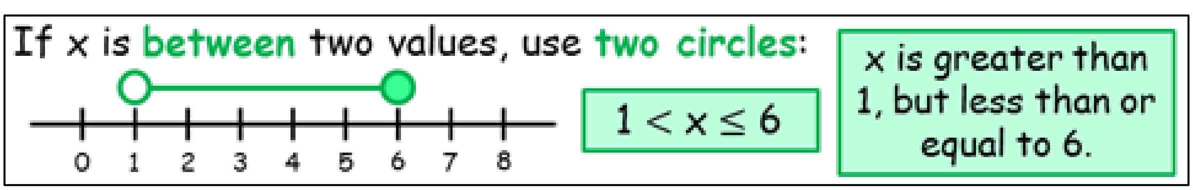



- [ ] a square bracket when we want to **include** the end value, or
- ( ) a round bracket when we **don't**

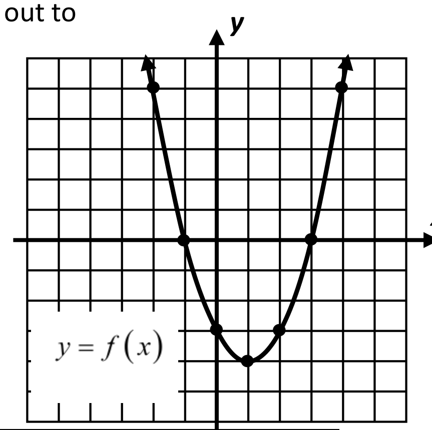
We can also show inequalities graphically:



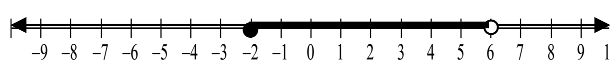
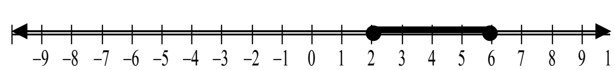



Some functions, defined with graphs or equations, have domains and ranges that stretch out to infinity. Consider the following exercise in which a standard parabola is graphed.




***5.*** The function $f(x) = x^2 - 2x - 3$ is graphed on the grid to the right. What would represent its domain and range written in interval notation?

| Symbol | Meaning | Open or Closed | Bracket or Parenthesis |
|---|---|---|---|
| $<$ | | | |
| $\leq$ | | | |
| $>$ | | | |
| $\geq$ | | | |

| | Inequality | Interval Notation | Graph |
|---|---|---|---|
| Ex. | $-3 \leq x < 5$ | [-3,5) |  |
| 1. | $x \leq 3$ | | |
| 2. | | $(-\infty,4)$ | |
| 3. | | |  |

# 1-2 FUNCTION COMPOSITIONS

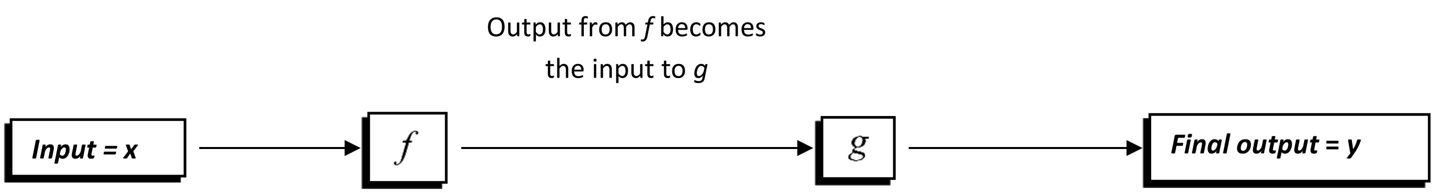
Since functions convert the value of an input variable into the value of an output variable, it stands to reason that this output could then be used as an input to a second function. This process is known as composition of functions, in other words, combining the action or rules of two functions.

***Exercise* #1:** A circular garden with a radius of 15 feet is to be covered with topsoil at a cost of \$1.25 per square foot of garden space.

(a) Determine the area of this garden to the nearest square foot.

(b) Using your answer from (a), calculate the cost of covering the garden with topsoil.

In this exercise, we see that the output of an area function is used as the input to a cost function. This idea can be generalized to generic functions, *f* and *g* as shown in the diagram below.




There are two notations that are used to indicate composition of two functions. These will be introduced in the next few exercises, both with equations and graphs.

***Exercise* #2:** Given $f(x) = x^2 - 5$ and $g(x) = 2x + 3$, find values for each of the following.

(a) $f(g(1)) =$

(b) $g(f(2)) =$

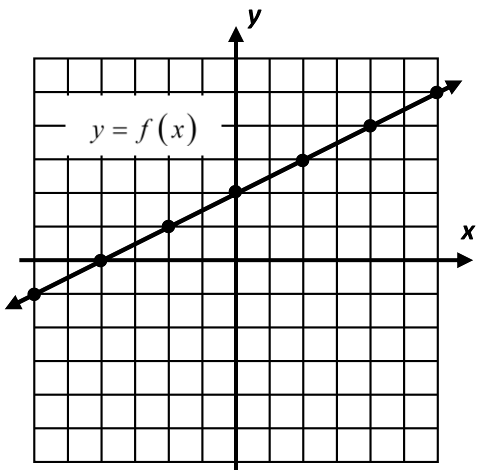
(c) $g(g(0)) =$

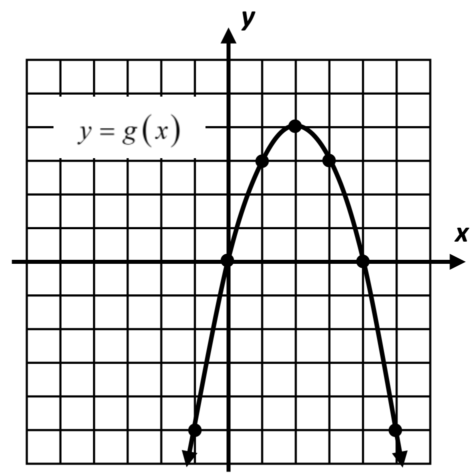
(d) $(f \circ g)(-2) =$

(e) $(g \circ f)(3) =$

(f) $(f \circ f)(-1) =$

***Exercise* #3:** The graphs below are of the functions $y = f(x)$ and $y = g(x)$. Evaluate each of the following questions based on these two graphs.




(a) $g(f(2)) =$

(b) $f(g(-1)) =$

(c) $g(g(1)) =$

(d) $(g \circ f)(-2) =$

(e) $(f \circ g)(0) =$

(f) $(f \circ f)(0) =$

On occasion, it is desirable to create a formula for the composition of two functions. We will see this facet of composition throughout the course as we study functions. The next two exercises illustrate the process of finding these equations with simple linear and quadratic functions.

***Exercise* #4:** Given the functions $f(x) = 3x - 2$ and $g(x) = 5x + 4$, determine formulas in simplest $y = ax + b$ form for:

(a) $f(g(x))$

(b) $g(f(x))$

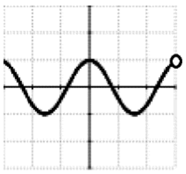
***Exercise* #5:** If $f(x) = x^2$ and $g(x) = x - 5$ then

$f(g(x)) =$

$g(f(x))$

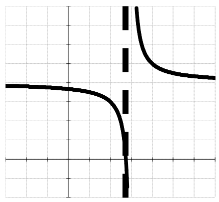
# 1-3 FEATURES OF GRAPHS

***For each graph below, state the domain and the range IN INTERVAL NOTATION***

1. 

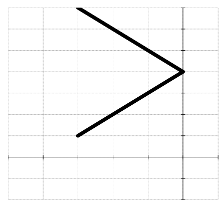
D: ________________

R: ________________

2. 

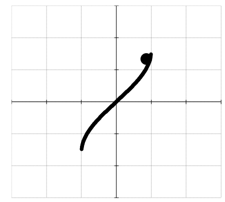
D: _______________

R: _______________

3. 

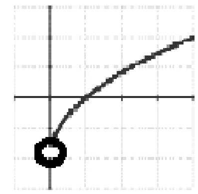
D: _______________

R: _______________

4. 

D: ______________

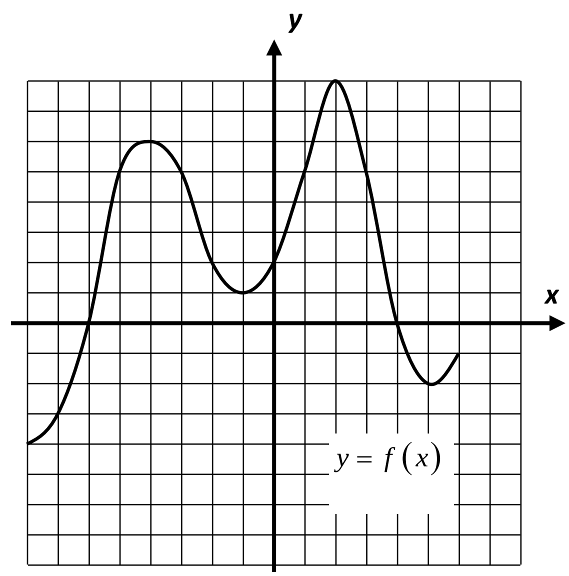
R: ______________

5. 

D: _______________

R: _______________

The graphs of functions have many key features whose terminology we will be using all year. It is important to master this terminology, most of which you learned in Common Core Algebra I.




***Exercise* #1:** The function $y = f(x)$ is shown graphed to the right. Answer the following questions based on this graph.

(a) State the *y*-intercept of the function.

(b) State the *x*-intercepts of the function. What is the alternative name that we give the *x*-intercepts?

(c) Over the interval $-1 < x < 2$ is $f(x)$ increasing or decreasing? How can you tell?

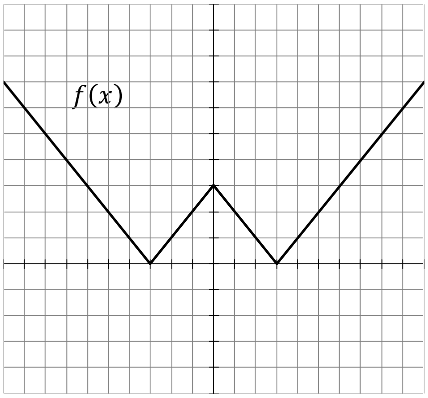
(d) Give the interval over which $f(x) > 0$. What is a quick way of seeing this visually?

(e) State all the *x*-coordinates of the relative maximums and relative minimums. Label each.

(f) What are the absolute maximum and minimum values of the function? Where do they occur?

(g) State the domain and range of $f(x)$ using interval notation.

Given $f(x)$ graphed below, determine each of the following:




y –intercept: X – intercept(s):

$f(-2) =$ $3f(6) - f(1) =$

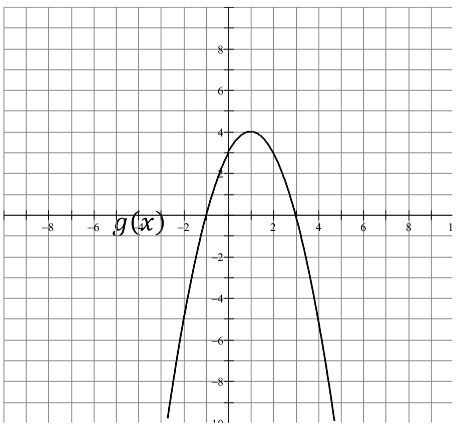
Circle all that are true:

$f(-2) = f(2)$ $-2f(-2) < -f(2)$

$f(-4) \leq \frac{f(7)}{2}$ $f(-4) > \frac{f(7)}{2}$

On what interval of x is f(x) increasing? On what interval of x is f(x) positive?

Given $g(x)$ graphed below, determine each of the following:




y –intercept: X – intercept(s):

Distance between the zeros:

$g(-2) =$ $3g(4) - g(1) =$

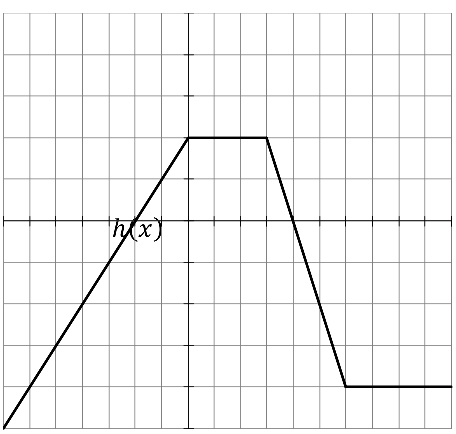
Circle all that are true:

$g(-2) = g(2)$ $-2g(-2) < -g(2)$

$g(-4) \leq \frac{g(7)}{2}$ $g(-4) > \frac{g(7)}{2}$

On what interval of x is g(x) decreasing? On what interval of x is g(x) positive?

Given $h(x)$ graphed below, determine each of the following:




y –intercept: X – intercept(s):

Distance between the zeros:

$h(-2) =$ $3h(4) - h(1) =$

Circle all that are true:

$f(-2) = h(2)$ $-2f(-2) < -h(2)$

$h(-4) \leq \frac{g(1)}{2}$ $g(-4) > \frac{h(7)}{2}$

On what interval of x is h(x) decreasing? On what interval of x is h(x) positive?

# REVIEW - LINEAR FUNCTIONS

**Slope-Intercept Form** of a Linear Equation

m= ___________ b= ____________

**Intercepts**

The ______________________________ of a line is the point at which the line crosses the y-axis.

The ______________________________ of a line is the point at which the line crosses the x-axis.

**Standard Form** of a Linear Equation:

**Ax + By = C** where A, B and C are integers.

To graph an equation in standard form, change it to ______________________________ form!

**Example :** Change to y-intercept form:

a) $6x + 8y = 24$

b) $2x - 5y = -20$

**Let's Graph!**: Graph the equation of **$3x - 4y = 12$ .**

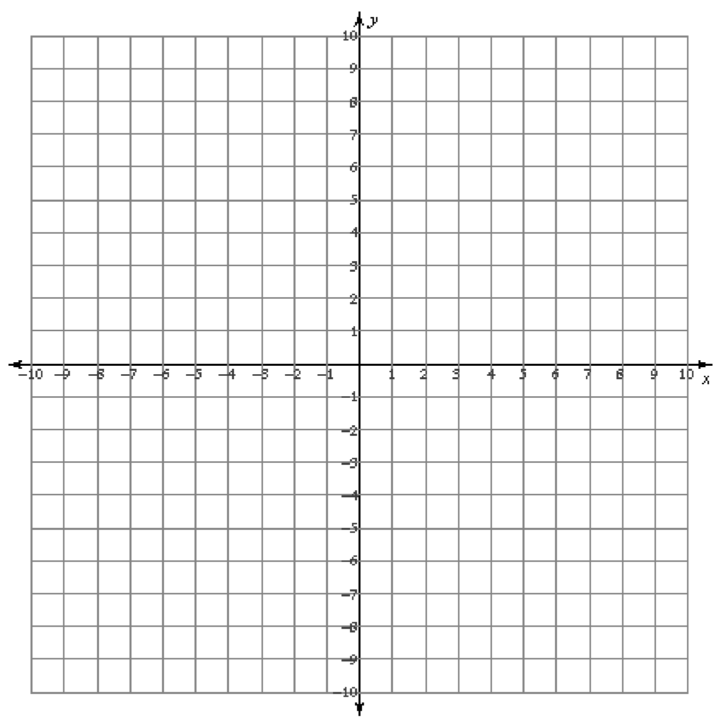
a. Complete the table.

| x | y |
|---|---|
| | |
| | |
| | |
| | |

b. Write in y-intercept form:__________

c. What is the slope:________

d. What is the y-intercept:__________



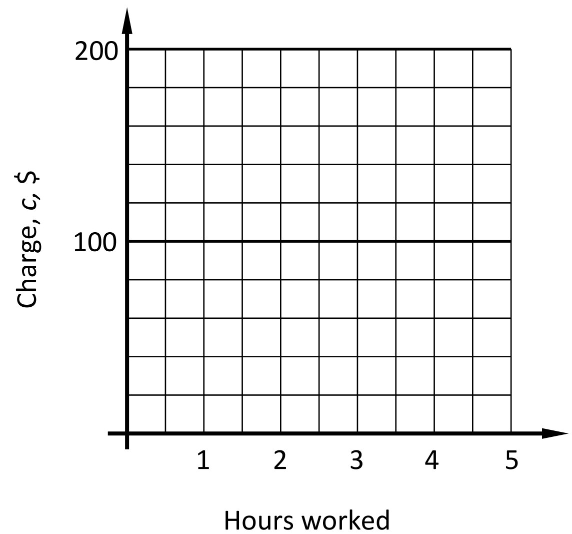
# 1-4 PIECEWISE STEP FUNCTIONS

Step functions, or ones whose outputs stay constant and then **jump** to a new constant value, are critical to a number of real world applications. Many times these types of functions arise in the areas of business.

***Exercise* #1:** An electrician works at a job site at a rate of $40 per hour or any portion of an hour. In other words, he will charge you $40 as soon as he comes up to the first hour, and then $40 for the second hour, etcetera.

(a) Graph the amount the electrician charges, *c*, in dollars as a function of the number of hours he works.

(b) How much does he charge for working 3.5 hours? Circle the point on the graph the shows this answer.



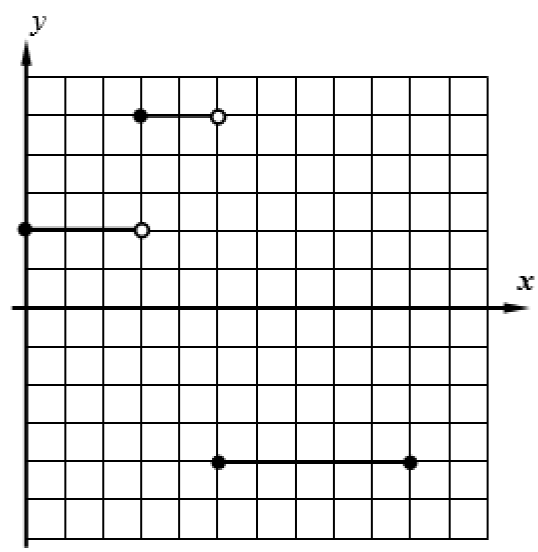

Step functions are rather simple because they consist of multiple **horizontal lines**. When reading their formula definitions, it is important to pay attention to the **domain intervals**.

***Exercise* #2:** A step function is defined using the piecewise formula $f\left(x\right)=\begin{cases} 2 & 0\le x<3 \\ 5 & 3\le x<5 \\ -4 & 5\le x\le 10 \end{cases}$.

(a) Evaluate the following:

$f\left(2.7\right)=$ $\qquad\qquad$ $f\left(5\right)=$

$f\left(3.5\right)=$ $\qquad\qquad$ $f\left(0\right)=$



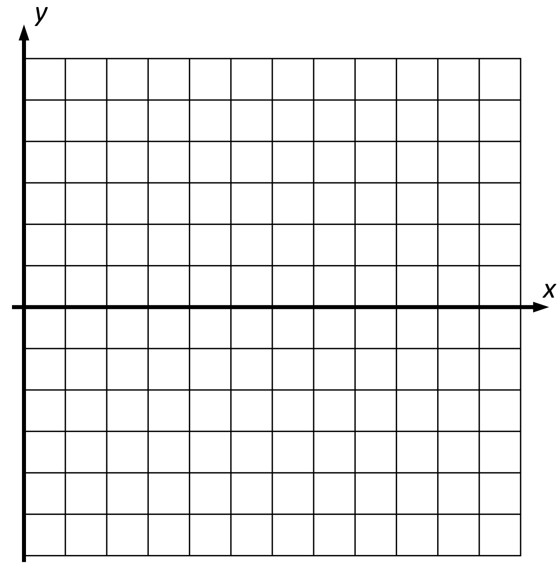

(b) State the domain and range of this function.

Domain: $\qquad\qquad$ Range:

(c) Using function notation and the function *f(x)*, write one equation that is true and two inequalities.

## WITHOUT CONTEXT

1. Consider the step function given by $f(x)=\begin{cases} 5 & 0\le x<4 \\ 1 & 4\le x<8 \\ -3 & 8\le x\le 12 \end{cases}$, which actually does a half-way decent job of modeling downward steps.



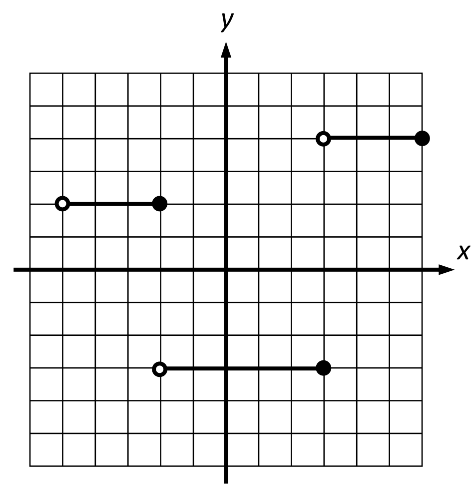

(a) Graph $f(x)$ on the grid provided.

(b) State the range of this function.

(c) Does $f(x)$ have any zeroes? Explain.

2. The step function $g(x)$ is shown on the grid to below. Answer the following questions.

(a) Evaluate each of the following:




$g(-4)=$ $\qquad\qquad$ $g(-2)=$

$g(2)=$ $\qquad\qquad$ $g(5)=$

(b) Ji Hwan states that the range of this function is $-3\le y\le 4$.
Is he correct? Why or why not.

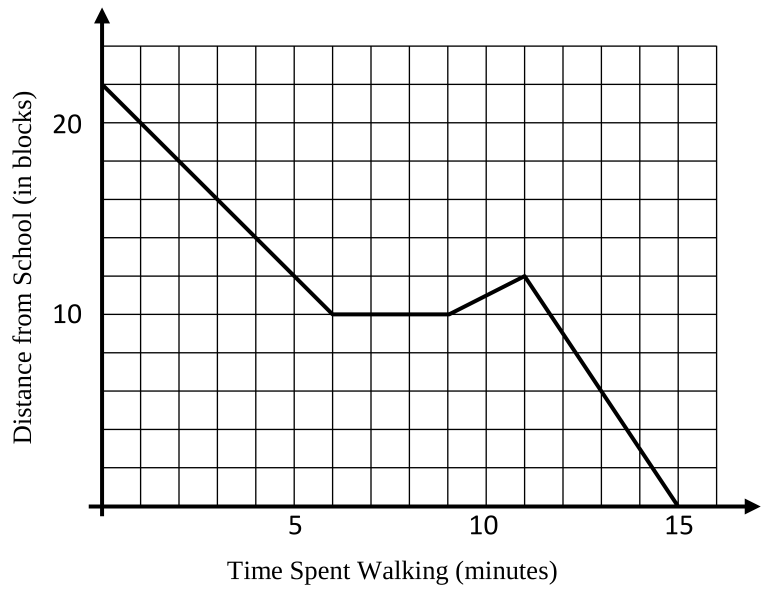
(c) Write an equation for this step function:

$$g(x)=\begin{cases} \\ \\ \\ \end{cases}$$

## PIECEWISE LINEAR FUNCTIONS

In today's lesson we will work specifically with **piecewise linear functions**, or those that are comprised of **linear segments**. These are particularly helpful in modeling certain situations, especially with **motion**. Functions are useful because they can often be used to **model** things that are happening in the real world. The next exercises illustrates a function given only in graphical form.

***Exercise* #1:** Charlene heads out to school by foot on a fine spring day. Her distance from school, in blocks, is given as a function of the time, in minutes, she has been walking. This function is represented by the graph given below.




(a) How far does Charlene start off from school?

(b) What is her distance from school after she has been walking for 4 minutes?

(c) After walking for six minutes, Charlene stops to look for her subway pass. How long does she stop for?

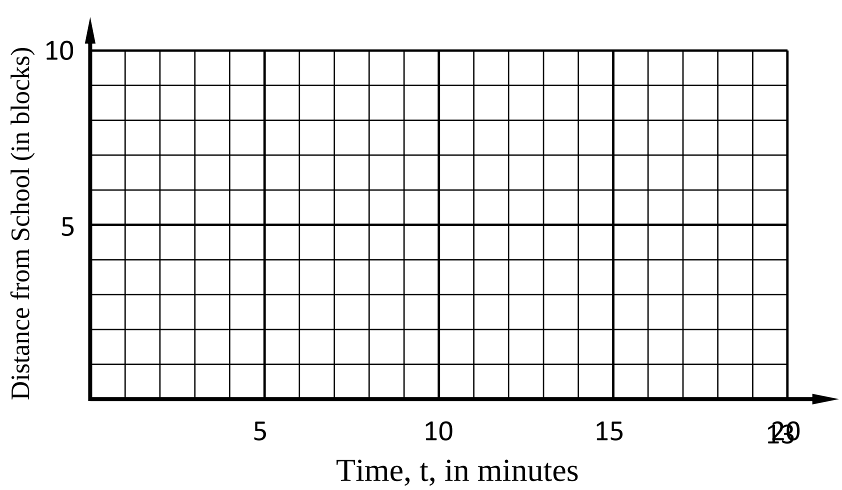
(d) Charlene then walks to a subway station before heading to school on the subway (a local). How many blocks did she walk to the subway?

(e) How long did it take for her to get to school once she got on the train?

***Exercise* #2:** Mateo is walking to school. It's a nice morning, so he is moving at a comfortable pace. After walking for 9 minutes, he is 6 blocks from home. He stops to answer a text on his phone from his mother. After 5 minutes standing still, he walks home quickly in 6 minutes to get a paper he forgot for school. We are going to model Mateo's distance from home, *D*, in blocks as a function of the time, *t*, in minutes since he left.

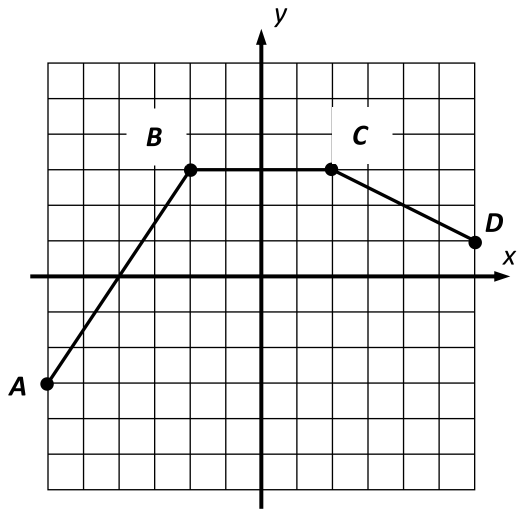
(a) Draw a graph of Mateo's distance from home on the grid provided.

(b) Determine a formula for the distance he is from home, *D*, over the time interval $9 \le t \le 14$.



Piecewise linear functions are more **complex function rules**. One way or another, though, they fit the standard definition of a function, i.e. for **every value in the domain (*x*)** there is only **one value in the range (*y*)**.

***Exercise* #3:** The piecewise linear function $f(x)$ is shown graphed below.




(a) Find the slope of each of the line segments:

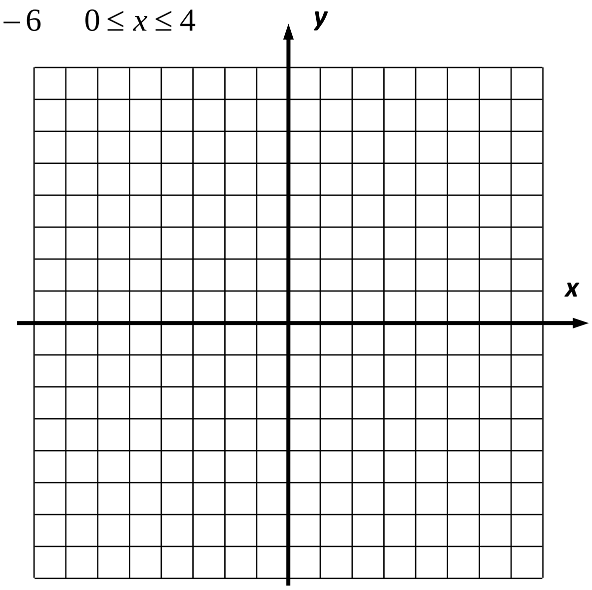
$\overline{AB}$:  $\overline{BC}$:  $\overline{CD}$:

(b) Now find the equation of the line that passes through each of the following pairs of points in $y = mx + b$ form where applicable. How can you find the *y*-intercepts by using the graph?

$\overleftrightarrow{AB}$:  $\overleftrightarrow{BC}$:  $\overleftrightarrow{CD}$:

(c) Write the formal piecewise definition for this function.

**Exercise 4:** On the graph below, sketch the function $h(x) = \begin{cases} -2x - 6 & -6 \le x < 0 \\ \frac{1}{2}x - 6 & 0 \le x \le 4 \end{cases}$

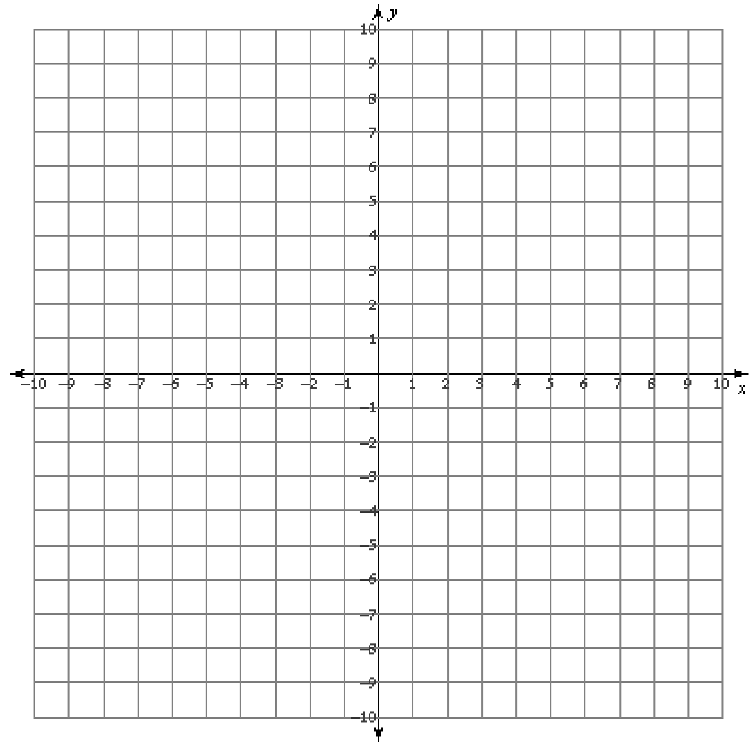



(a) Graph $h(x)$ on the grid.

(b) State the range of $h(x)$.

(c) What values of *x* solve $h(x) = 0$?

**PIECEWISE FUNCTIONS!**



### Ex. 1: Graphing and Evaluating Piecewise Functions

$$f(x) = \begin{cases} x + 5 & \text{if } x \le -2 \\ 2x - 1 & \text{if } -2 < x \le 2 \\ 3x - 9 & \text{if } x > 2 \end{cases}$$

Now Evaluate!

a. $f(-10) =$

c. $f(0) =$

e. $f(-21) =$

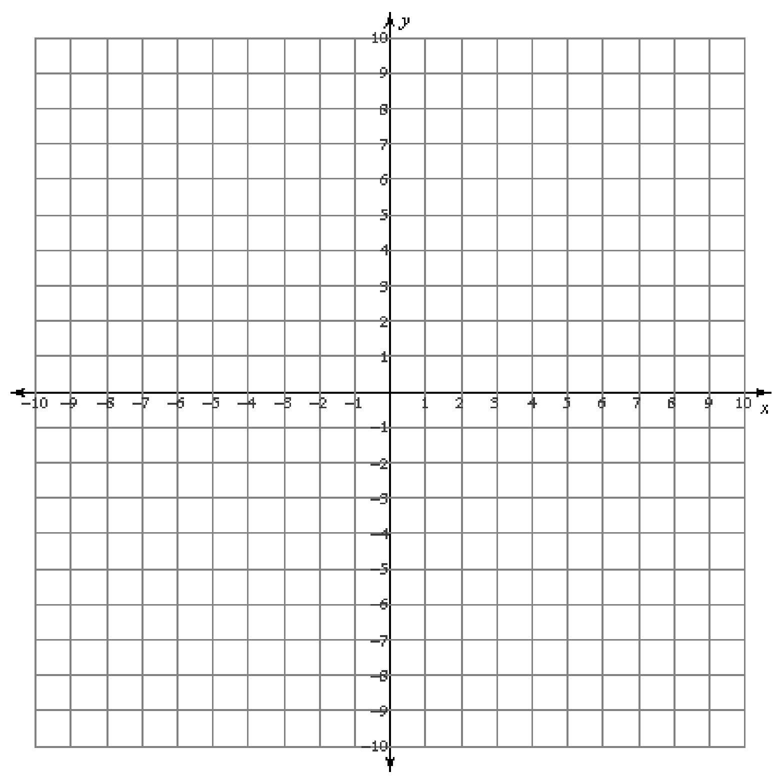
b. $f(6) =$

d. $f(-2) =$

f. $f(13) =$

### Ex 2: Graph the Piecewise Function

$$g(x) = \begin{cases} 2x + 3 & \text{if } -3 < x \le 1 \\ x & \text{if } 1 < x \le 5 \\ -\frac{1}{2}x + 8 & \text{if } x > 6 \end{cases}$$



a. $2g(8) - 3g(1)$

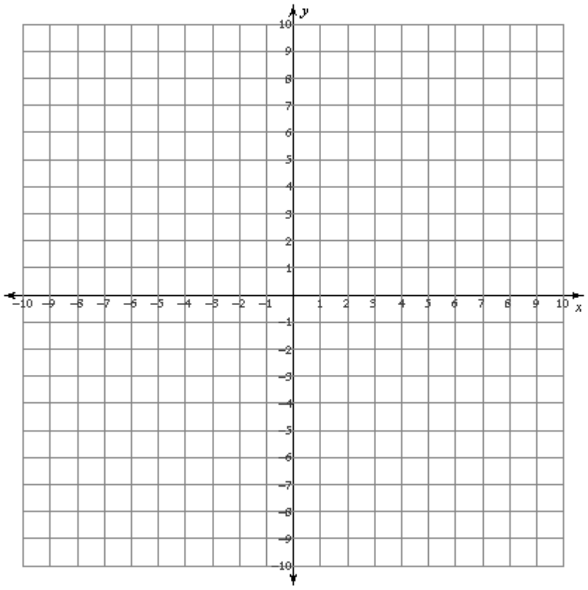
b. $\frac{1}{2}\, g(4) + 4g(0)$

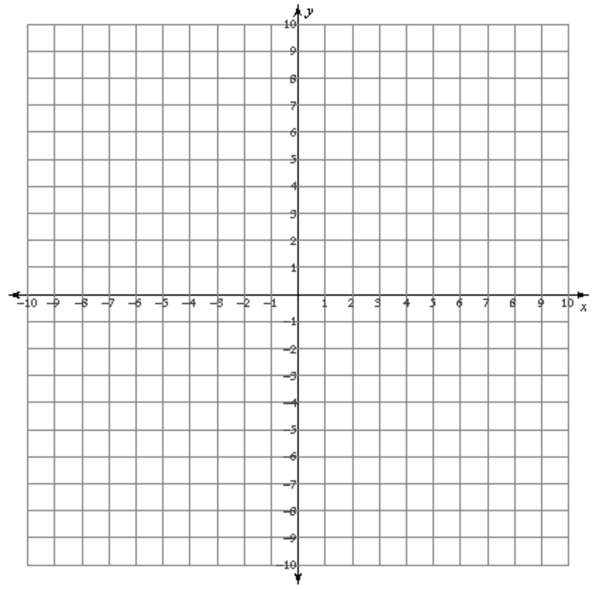
# Piecewise Function Practice

Graph each of the following Piecewise Functions:

$f(x) = \begin{cases} 2 & x < -1 \\ -x - 1 & x \geq -1 \end{cases}$

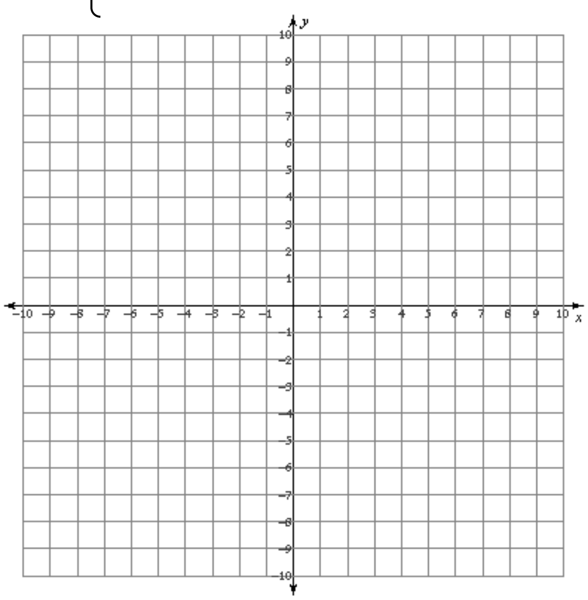
$g(x) = \begin{cases} -2x - 3 & x < 1 \\ x - 4 & x > 1 \end{cases}$

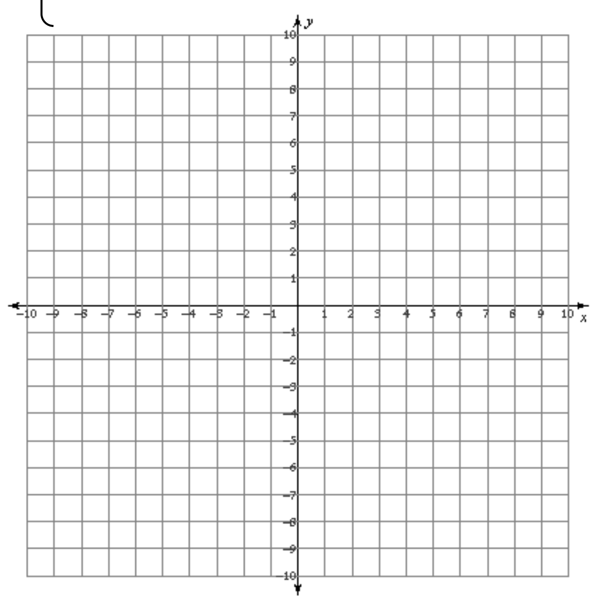




$h(x) = \begin{cases} x & x < -3 \\ 2 & -3 \leq x < 1 \\ -2\text{x} + 2 & x \geq 1 \end{cases}$

$m(x) = \begin{cases} 4 & 0 \leq x < 2 \\ 5 & 2 \leq x < 4 \\ 6 & 4 \leq x < 6 \end{cases}$





$f(-1) =$ ______ $g(1) =$ ______ $h(-3) =$ ______ $m(2) =$ ______

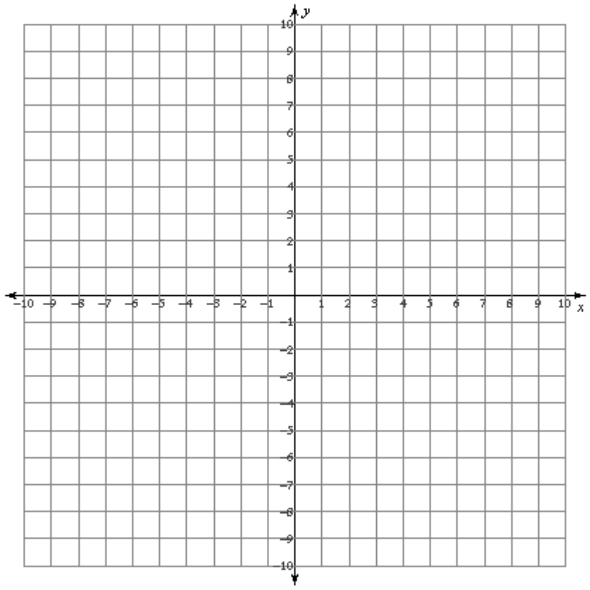
$-2f(-1) =$ ______ $g(1) - 4 =$ ______ $h(-3)/2 =$ ______ $-9m(2) =$ ______

# 1-5 Inverse Functions

Graph $g(x) = \frac{1}{2}x + 4$ and fill in the table.

Graph $f(x) = 2x - 8$ and fill in the table

| x | g(x) |
|---|---|
| -10 | |
| -8 | |
| -6 | |
| -4 | |
| -2 | |
| 0 | |
| 2 | |
| 4 | |
| 6 | |
| 8 | |



| x | f(x) |
|---|---|
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |

1. What do you notice about the ordered pairs in each function? Is there a relationship between f(x) and g(x)?

2. Graph the line y = x on the same graph. Describe your lines in relationship to this one.

To find the _________ of a function:

1.

2.

Symbol:

$f^{-1}$

Find the $f^{-1}$

a) f(x)= {(4, -3), (2, 8), (3, -8) (0, -3)}

b) y = 3x-7

c) $f(x) = -\frac{1}{2}x + 3$

Note: Domain of a function = ____________________________ of its inverse

Range of a function = ____________________________ of its inverse!

Practice: Find the inverse of the following functions:

1) $f(x) = 3x + 5$

2) $g(x) = \frac{1}{3}x - 12$

Domain:

Range:

Domain:

Range:

Application:

3. Mr. Desmond decided to go to the North Carolina State Fair. Entry into the fair was $8, and then each ride was an additional $2.

   a) Write a function f(x) to describe the total cost of going to the fair, with x representing the amount of rides purchased.

   b) Find f(3) and describe its meaning in context.

   c) If Mr. Desmond paid $20 at the fair, how many rides did he ride? Show your algebra!

   d) Write a function f-1(x) to describe the number of rides purchased given the total cost x.

   e) If you went to the State Fair and spent $40, how many rides did you ride?

4. A car traveling at a constant speed of 58 miles per hour has a distance of *y*-miles from Poughkeepsie, NY, given by the equation $y = 58x + 24$, where *x* represents the time in hours that the car has been traveling.

(a) Find the equation of the inverse of this linear function in $y = \frac{x-a}{b}$ form.

(b) Evaluate the function you found in part (a) for an input of $x = 227$.

(c) Give a physical interpretation of the answer you found in part (b). Consider what the input and output of the inverse represent in order to answer this question.