Name: ______________________________ Date: ________________

SOLVING EXPONENTIAL EQUATIONS USING LOGARITHMS
COMMON CORE ALGEBRA II

Earlier in this unit, we used the **Method of Common Bases** to solve exponential equations. This technique is quite limited, however, because it requires the two sides of the equation to be expressed using the same base. A more general method utilizes our calculators and the third logarithm law:

THE THIRD LOGARITHM LAW

$$\log_b\left(a^x\right) = x\log_b a$$

***Exercise* #1:** Solve: $4^x = 8$ using (a) common bases and (b) the logarithm law shown above.

(a) Method of Common Bases

(b) Logarithm Approach

The beauty of this logarithm law is that it removes the variable from the exponent. This law, in combination with the logarithm base 10, the **common log**, allows us to solve almost any exponential equation using calculator technology.

***Exercise* #2:** Solve each of the following equations for the value of *x*. Round your answers to the nearest *hundredth*.

(a) $5^x = 18$

(b) $4^x = 100$

(c) $2^x = 1560$

These equations can become more complicated, but each and every time we will use the logarithm law to transform an exponential equation into one that is more familiar (linear only for now)

***Exercise* #3:** Solve each of the following equations for *x*. Round your answers to the nearest *hundredth*.

(a) $6^{x+3} = 50$

(b) $(1.03)^{\frac{x}{2}-5} = 2$

Now that we are familiar with this method, we can revisit some of our exponential models from earlier in the unit. Recall that for an exponential function that is growing:

> If quantity Q is known to increase by a fixed percentage p, in decimal form, then Q can be modeled by
>
> $$Q(t)=Q_0(1+p)^t$$
>
> where Q_0 represents the amount of Q present at $t=0$ and t represents time.

***Exercise* #4:** A biologist is modeling the population of bats on a tropical island. When he first starts observing them, there are 104 bats. The biologist believes that the bat population is growing at a rate of 3% per year.

(a) Write an equation for the number of bats, $B(t)$, as a function of the number of years, t, since the biologist started observing them.

(b) Using your equation from part (a), algebraically determine the number of years it will take for the bat population to reach 200. Round your answer to the nearest year.

***Exercise* #5:** A stock has been declining in price at a steady pace of 5% per week. If the stock started at a price of \$22.50 per share, determine algebraically the number of weeks it will take for the price to reach \$10.00. Round your answer to the nearest week.

As a final discussion, we return to evaluating logarithms using our calculator. Many modern calculators can find a logarithm of any base. Some still only have the common log (base 10) and another that we will soon see. But, we can still express our answers in terms of logarithms.

***Exercise* #6:** Find the solution to each of the following exponential equations in terms of a logarithm with the same base as the exponential equation.

(a) $4(2)^x-3=17$

(b) $17(5)^{x/3}=4$

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5-8 Solving Exponential Equations Using Logarithms
Math III Homework

Fluency

1. Which of the following values, to the nearest *hundredth*, solves: $7^x = 500$?

   (1) 3.19 (3) 2.74

   (2) 3.83 (4) 2.17

   ____

2. The solution to $2^{x/3} = 52$, to the nearest *tenth*, is which of the following?

   (1) 7.3 (3) 11.4

   (2) 9.1 (4) 17.1

   ____

3. To the nearest *hundredth*, the value of *x* that solves $5^{x-4} = 275$ is

   (1) 6.73 (3) 8.17

   (2) 5.74 (4) 7.49

   ____

4. Solve each of the following exponential equations. Round each of your answers to the nearest *hundredth*.

   (a) $9^{x-3} = 250$ (b) $50(2)^x = 1000$ (c) $5^{x/10} = 35$

5. Solve each of the following exponential equations. Be careful with your use of parentheses. Express each answer to the nearest *hundredth*.

   (a) $6^{2x-5} = 300$ (b) $\left(1/2\right)^{\frac{x}{3}+1} = 1/6$ (c) $500(1.02)^{\frac{x}{12}} = 2300$

APPLICATIONS

6. The population of Red Hook is growing at a rate of 3.5% per year. If its current population is 12,500, in how many years will the population exceed 20,000? Round your answer to the nearest year. Only an *algebraic* solution is acceptable.

7. A radioactive substance is decaying such that 2% of its mass is lost every year. Originally there were 50 kilograms of the substance present.

   (a) Write an equation for the amount, A, of the substance left after t-years.

   (b) Find the amount of time that it takes for only half of the initial amount to remain. Round your answer to the nearest tenth of a year.

REASONING

8. If a population doubles every 5 years, how many years will it take for the population to increase by 10 times its original amount?

   First: If the population gets multiplied by 2 every 5 years, what does it get multiplied by each year? Use this to help you answer the question.

9. Find the solution to the general exponential equation $a(b)^{cx} = d$, in terms of the constants a, c, d and the logarithm of base b. Think about reversing the order of operations in order to solve for x.