Name: ____________________________________ Date: ____________________

# UNIT #2 – TRANSFORMATIONS OF FUNCTIONS

**Part I Questions**

1. The quadratic function $f(x)$ has a turning point at $(5,-8)$. If $g(x)=f(x+7)-3$, then at which of the following does $g(x)$ have a turning point?

(1) $(-2,-11)$ (3) $(-7,-3)$

(2) $(12,-11)$ (4) $(12,-5)$

**The structure of this transformation indicates that the graph of *f* has been shifted left 7 units due to 7 being added to the input. And then the graph has been shifted down 3 units due to 3 being subtracted from the function. Hence:**

$$(5,-8)\to(5-7,-8-3)$$
$$\to(-2,-11)$$

**(1)**

2. Where does the absolute value function $y=\frac{1}{2}|x-8|+3$ have a turning point?

(1) $(-4,3)$ (3) $(8,3)$

(2) $(4,-3)$ (4) $(8,-3)$

**This function is a shift of 8 right and 3 up of the more simple function $y=\frac{1}{2}|x|$, which has a turning point at $(0,0)$. Thus, it will have a turning point at $(8,3)$**

**(3)**

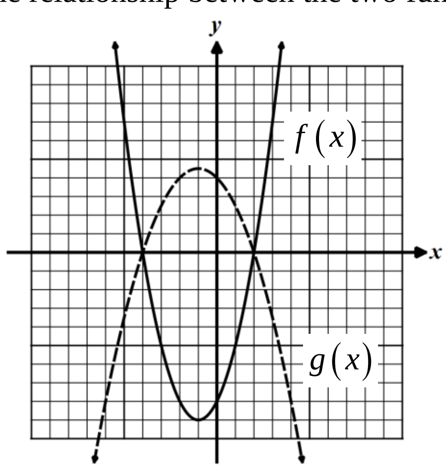
3. The function $f(x)$ is shown below graphed in solid while the function $g(x)$ is shown dashed. Which of the following equations describes the relationship between the two functions?

(1) $g(x)=f(x)-6$

(2) $g(x)=-\frac{1}{2}f(x)$

(3) $g(x)=2f(x)$

(4) $g(x)=f\left(\frac{1}{2}x\right)$




**The graph of $f(x)$ has been both reflected and compressed vertically to produce the graph of $g(x)$. To do this, one has to multiple the entire function by a negative number between 0 and -1.**

**(2)**

4. Given that the function $y=x^2+6x-27$ has $x$-intercepts at $x=-9$ and $x=3$, where does the function $y=(3x)^2+6(3x)-27$ have $x$-intercepts?

(1) $x=\pm 6$ (3) $x=-27$ and $x=9$

(2) $x=-12$ and $x=0$ (4) $x=-3$ and $x=1$

**Since $x=-9$ is an $x$-intercept of this function we know:**

$$(-9)^2+6(-9)-27=0$$

**But, this allows us to simply solve the following.**

$$3x=-9$$
$$x=-3$$

$$3x=3$$
$$x=1$$

**(4)**

5. If the point $(-3, 7)$ lies on the graph of $f(x)$, then which of the following points must lie on the graph of $y = 5f(x) - 20$?

(1) $(-15, -13)$ (3) $(2, -13)$

(2) $(-3, 15)$ (4) $(1, 25)$

**$f(-3) = 7$ so:**

$$\begin{aligned} y &= 5f(-3) - 20 \\ &= 5(7) - 20 \\ &= 35 - 20 \\ &= 15 \end{aligned}$$

**(2)**

6. The range of the function $f(x)$ is $-4 \le y \le 10$. If $g(x) = -f(x) + 3$ then which of the following is the range for $g(x)$?

(1) $-7 \le y \le 7$ (3) $-13 \le y \le 1$

(2) $5 \le y \le 15$ (4) $-3 \le y \le 8$

**The structure of $g(x)$ indicates that its graph is a reflection of $f(x)$ in the x-axis followed by an upward shift of 3 units. Thus, the range would be transformed as:**

$$[-4, 10] \xrightarrow{r_{x-axis}} [-10, 4] \xrightarrow{T_{0,3}} [-7, 7]$$

**(1)**

7. The trinomial $4x^2 - 3x - 10$ can be written equivalently as

(1) $(2x-5)(2x+2)$ (3) $(4x+1)(x-10)$

(2) $(2x-1)(x+10)$ (4) $(4x+5)(x-2)$

**Multiply each of the following pairs of binomials to see which is equivalent to the original trinomial. Choice (4) gives:**

$$\begin{aligned} (4x+5)(x-2) &= 4x^2 - 8x + 5x - 10 \\ &= 4x^2 - 3x - 10 \end{aligned}$$

**(4)**

8. The cubic polynomial $3x^3 + 5x^2 + 12x + 20$ can be factored as

(1) $(3x+5)(x+2)^2$ (3) $(x+5)(3x+2)$

(2) $(3x+5)(x^2+4)$ (4) $(x+5)(x-2)(x+2)$

$$\begin{aligned} &= x^2(3x+5) + 4(3x+5) \\ &= (3x+5)(x^2+4) \end{aligned}$$

**(2)**

9. The equation $5x(x-7)^2(3x+2) = 0$ has a solution set of

(1) $\left\{-5, -\frac{2}{3}, \pm 7\right\}$ (3) $\left\{-\frac{2}{3}, 0, 7\right\}$

(2) $\{-5, -2, 7\}$ (4) $\{-2, 0, \pm 7\}$

$$\begin{aligned} 5x &= 0 \\ x &= 0 \end{aligned}$$

$$\begin{aligned} (x-7)^2 &= 0 \\ x - 7 &= \sqrt{0} \\ x - 7 &= 0 \\ x &= 7 \end{aligned}$$

$$\begin{aligned} 3x + 2 &= 0 \\ 3x &= -2 \\ x &= -\frac{2}{3} \end{aligned}$$

**(3)**

10. The quadratic function $f(x)=10x^2+11x-6$ has one zero at $x=-\frac{3}{2}$. At which of the following $x$-values is its other zero?

(1) $x=6$

(2) $x=\frac{2}{5}$

(3) $x=\frac{1}{6}$

(4) $x=-4$

**Since $x=-\frac{3}{2}$ is a zero, then $2x+3$ must be a factor of this polynomial. But, this means it must factor as:**

$$(2x+3)(5x-2)$$

**Which means its other zero must be $x=\frac{2}{5}$**

**(2)**

11. The parabola $y=3x^2-24x+55$ can be written in the form

(1) $y=3(x-2)^2+2$

(2) $y=3(x-8)^2+55$

(3) $y=3(x+2)^2-11$

(4) $y=3(x-4)^2+7$

$$y=3(x^2-8x)+55$$
$$y=3(x^2-8x+16)-3(16)+55$$
$$y=3(x-4)^2-48+55$$
$$y=3(x-4)^2+7$$

**(4)**

12.

Which equation below represents the graph shown?

(1) $y=\sqrt{x-2}-5$

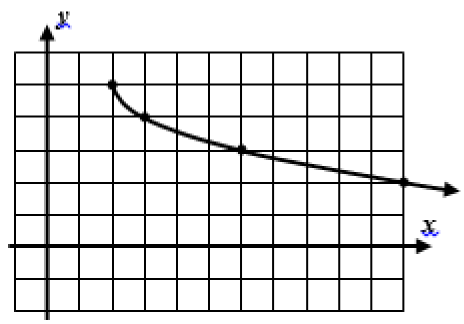
(2) $y=-\sqrt{x+2}+5$

(3) $y=-\sqrt{x-2}+5$

(4) $y=\sqrt{x+2}+5$

**This graph is a reflection of the graph of $y=\sqrt{x}$ followed by a rightward shift by 2 units and upward shift by 5 units. The new equation is thus:**

$$y=-\sqrt{x-2}+5$$




**(3)**

## Free Response

13. For the function $f(x)$ it is known that $(-12,4)$ lies on the function. A second function, $g(x)$, is definedby the formula $g(x)=f(2x)-3$.

Describe the transformations that occur to the graph of $f$ in order to produce the graph of $g$.

**There are two transformations that occur:**

**1. A horizontal compression by a factor of 2 (or if you prefer a factor of 1/2)**

**2. A vertical shift by 3 units downward.**

Based on the fact that the point $(-12,4)$ lies on $f(x)$, what point must lie on $g(x)$?

$$(-12,4)\to\left(\frac{-12}{2},4\right)\to(-6,4)$$

$$(-6,4)\to(-6,4-3)\to(-6,1)$$

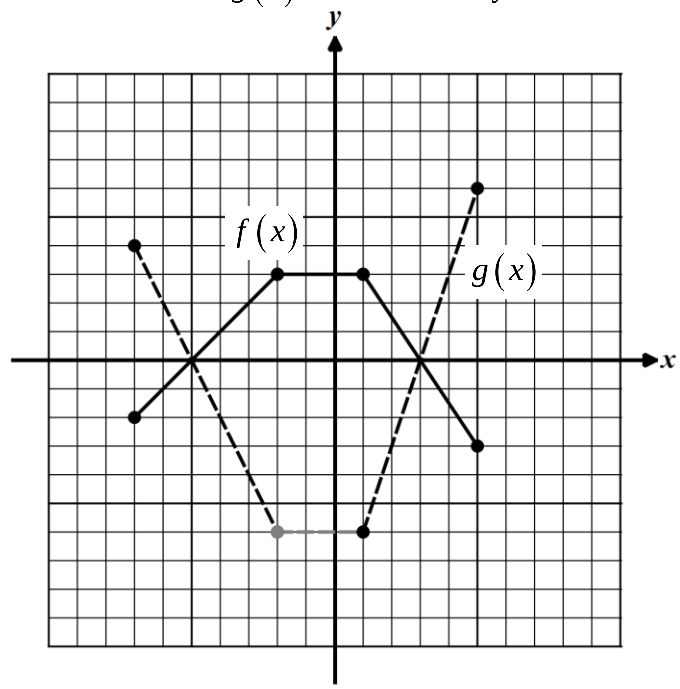
**Thus the point $(-6,1)$**

**Note that we can verify our answer by evaluating $g(-6)$:**

$$g(-6)=f(2\cdot-6)-3=f(-12)-3$$
$$=4-3=1$$

14. The graph of the function $f(x)$ is shown below. The function $g(x)$ is defined by the formula $g(x) = -2f(x)$ for all values of $x$.

Produce the graph of $g$ on the same grid.




**$(-7,-2) \to (-7,4)$**
**$(-2,3) \to (-2,-6)$**
**$(1,3) \to (1,-6)$**
**$(5,-3) \to (5,6)$**

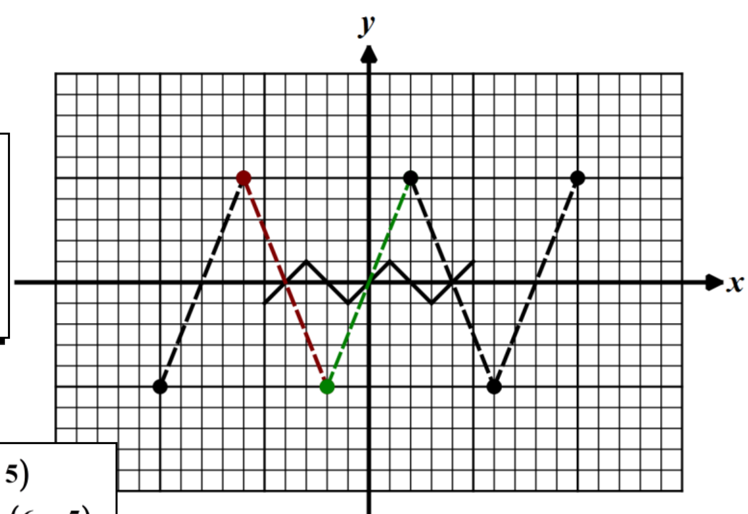
Solve the equation $f(x) = g(x)$ for all values of $x$.

**To solve any equation graphically, simply find the *x*-values where the outputs (*y*-values) are the same. In other words, give the *x*-values of the intersection points:**

**$x = -5$ and $x = 3$**

15. The graph of $f(x)$ is shown below. The function $g(x)$ is defined by $g(x) = 5f\left(\frac{x}{2}\right)$.

Explain the transformations that will transform the graph of $f(x)$ into the graph of $g(x)$ and then produce it on the same grid.




**Two transformations have occurred to *f* in order to produce the graph of *g*. These can occur in either order:**

**1. A horizontal stretch by a factor of 2.**

**2. A vertical stretch by a factor of 5.**

**$(-5,-1) \to (-10,-1) \to (-10,-5)$**
**$(-3,1) \to (-6,1) \to (-6,5)$**
**$(-1,-1) \to (-2,-1) \to (-2,-5)$**

**$(1,1) \to (2,1) \to (2,5)$**
**$(3,-1) \to (6,-1) \to (6,-5)$**
**$(5,1) \to (10,1) \to (10,5)$**

16. Given the parabola $f(x) = -(x-8)^2 + 5$, describe three transformations which would transform the graph of $y = x^2$ into the graph of $f(x)$. Give both the transformations and the order.

**There are multiple orders of transformations. We will present two correct options.**

**Option #1:**

**1. A horizontal shift right by 8 units.**

**2. A reflection in the *x*-axis.**

**3. A vertical shift upwards of 5 units.**

**Option #2:**

**1. A reflection in the *x*-axis.**

**2. A horizontal shift right by 8 units.**

**3. A vertical shift upwards of 5 units.**

17. Describe the difference between the transformations $f(-x)$ and $-f(x)$ on the graph of $f(x)$.

**The transformation $f(-x)$ will reflect the graph of $f(x)$ in the $y$-axis.**

**The transformation $-f(x)$ will reflect the graph of $f(x)$ in the $x$-axis.**

18. Factor the expression below completely.

$8x^2+12x-8$

$$= 4(2x^2+3x-2)$$
$$= 4(2x-1)(x+2)$$

$12x^3+20x^2-3x-5$

$$= 4x^2(3x+5)-1(3x+5)$$
$$= (3x+5)(4x^2-1)$$
$$= (3x+5)(2x+1)(2x-1)$$

$27x^3-y^3$

$$(3x-y)(9x^2+3xy+y^2)$$

19. Shana believes one of the two binomial factors of $12x^2+35x+8$ is $3x+2$. Is she correct? Explain your answer.

**If $(3x+2)$ is one of the two factors, then:**
$$12x^2+35x+8=(3x+2)(4x+4)$$

**The second factor would have to be $4x+4$ in order for the quadratic term of $12x^2$ and the constant term of 8 match.**

**But:**
$$(3x+2)(4x+4)=12x^2+8x+12x+8$$
$$=12x^2+20x+8$$

**Although, the quadratic and constant terms match, the linear term of $20x$ does not match the correct term of $35x$.**

20. Find each of the following cube roots without the use of your calculator. Justify your answer based on a multiplication statement.

(a) $\sqrt[3]{8}$

$= 2$

$2\cdot2\cdot2=2^3=8$

(b) $\sqrt[3]{-1}$

$= -1$

$-1\cdot-1\cdot-1=(-1)^3=-1$

(c) $\sqrt[3]{125}$

$= 5$

$5\cdot5\cdot5=5^3=125$

(d) $\sqrt[3]{0}$

$= 0$

$0\cdot0\cdot0=0^3=0$

(e) $\sqrt[3]{-8}$

$= -2$

$-2\cdot-2\cdot-2=(-2)^3=-8$

(f) $\sqrt[3]{27}$

$= 3$

$3\cdot3\cdot3=3^3=27$

(g) $\sqrt[3]{\frac{1}{64}}$

$= \frac{1}{4}$

$\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}=\left(\frac{1}{4}\right)^3=\frac{1}{64}$

(h) $\sqrt[3]{-\frac{1}{1000}}$

$= -\frac{1}{10}$

$-\frac{1}{10}\cdot-\frac{1}{10}\cdot-\frac{1}{10}=\left(-\frac{1}{10}\right)^3=-\frac{1}{1000}$